Sponsors:
ASA - Automata, Semigroups and Applications (PTDC/MAT/65481/2006)
CMUP - Centro de Matemática da Universidade do Porto

James Mitchell *CAUL and U. St Andrews*

COMPUTING AUTOMORPHISMS OF SEMIGROUPS

In this talk I will describe an algorithm for computing the automorphism group of an arbitrary finite semigroup, and demonstrate the implementation of this algorithm in gap.

Vítor H. Fernandes *Universidade Nova de Lisboa, CAUL*

ON THE RANKS OF SOME TRANSFORMATION SEMIGROUPS ON A FINITE SET

In this talk we present the ranks of several transformation semigroups on a finite set. Namely, we consider semigroups of transformations with restricted ranges and semigroups of transformations that preserve an equivalence relation and the order or the orientation. All these results were obtained, decisively, by working examples in GAP.

Manuel Delgado *CMUP and U. Porto*

SOME PROBLEMS ON NUMERICAL SEMIGROUPS

A numerical semigroup is a co-finite submonoid of the additive submonoid of non-negative integers. The number of positive integers not belonging to the semigroup is said to be the gender of the semigroup. During my talk I shall refer some problems on numerical semigroups in which I got interested recently. I shall start referring the classical Frobenius' problem (obtaining a formula in terms of generators to the largest integer that does not belong to the numerical semigroup). I shall then refer the counting of numerical semigroups (by gender) and the possible construction of a database of numerical semigroups. Then I shall mention the calculation of Feng Rao numbers. The interest in computing these numbers comes from the importance of the Feng Rao distance in Algebraic Geometry Codes.

Karl Auinger *U. Vienna*

THE FINITE BASIS PROBLEM FOR PARTITION SEMIGROUPS

I shall recall the definition of the partition semigroups $\mathfrak{C}_n, \mathfrak{B}_n, \mathfrak{A}_n, \mathfrak{I}_n$ (and others) and review some basic facts. Then I will present some results concerning the finite basis problem for some of these. This problem is considered with respect to the signature $\{\cdot, *\}$ where $*$ denotes the natural involution these semigroups are equipped with.

Emanuele Rodaro *CMUP*

ALGORITHMIC PROBLEMS OF AMALGAMS OF FINITE INVERSE SEMIGROUPS

Let S_1, S_2 be two inverse semigroups and let U be a common inverse subsemigroup with $\omega_i : U \hookrightarrow S_i$ for $i = 1, 2$. Suppose that $S_i = Inv\langle X_i | R_i \rangle$ with $X_1 \cap X_2 = \emptyset$, the free product with amalgamation amalgamating U is the inverse semigroup:

$$S_1 *_U S_2 = Inv\langle X_1, X_2 | R_1, R_2, R_W \rangle$$

where $R_W = \{(\omega_1(u), \omega_2(u)) : u \in U\}$.

We prove that the word problem and the problem of checking wether an amalgam of finite inverse semigroup contains a bicyclic monoid is a decidable problem.

Enric Ventura *U. Politècnica de Catalunya*

ENDOFIXEDNESS AND COMPUTATION OF FIXED CLOSURES IN FREE GROUPS

Let F be a finitely generated free group. We present an algorithm such that, given a subgroup H<F, decides whether H is the fixed subgroup of some family of automorphisms, or family of endomorphisms of F and, in the affirmative case, finds such a family. The given procedure, in fact, computes the auto-fixed and endo-fixed closures of H. The algorithm combines both combinatorial and geometric methods. These results are in contrast with the fact that the same questions for a single automorphism, or endomorphism, remain open.

Pedro Silva *CMUP and U. Porto*

FINITE IDEMPOTENT INVERSE MONOID PRESENTATIONS

The word problem for finite idempotent inverse monoid presentations was solved in 1987 by Margolis and Meakin, an alternative solution was proposed in 1990 by the author, and Diekert, Lohrey and Ondrusch achieved linear time solutions in 2006. Let M be an inverse monoid defined by such a presentation. Among other results, we prove that:

(1) M has finitely many D-classes with nontrivial groups, and their Schützenberger graphs can be effectively computed;
(2) M is semisimple if and only if every endomorphism of a Schützenberger graph is an automorphism;
(3) it is decidable whether or not M is combinatorial, semisimple, fundamental, or has some infinite D-class;
(4) if μ denotes the greatest idempotent-separating congruence on M, then the word problem is decidable for M/μ.

We also give a new elementary proof for the word problem for M.

Alan Cain *CMUP*

COMPUTING WITH AUTOMATIC STRUCTURES FOR SEMIGROUPS

From an algorithmic perspective, there are two main classes of problem for automatic semigroups: those where an automatic structure is the goal, and those where it is the starting-point. In the first class, the main problem is how to algorithmically obtain an automatic structure for a semigroup given some other description, such as a presentation. In the second class, are the problems of deciding various properties of an automatic semigroup, given an automatic structure. This talk will survey both types of problem, with a particular focus on recent work on computing automatic structures for monoids presented by confluent rewriting systems and their submonoids.

Jorge Almeida *CMUP and U. Porto*

SOME RECENT DEVELOPMENTS ON BRZOZOWSKI'S HIERARCHY OF STAR-FREE LANGUAGES

Words can be viewed as (order-linear) models of a first-order logical language so that formal languages become sets of models. The axiomatizability of such languages in the first-order theory turns out to be also equivalent to star-freeness (R. McNaughton and S. Pappert, 1971). Moreover, both the regular expression and the logical approach suggest a natural hierarchy of star-free languages: counting how many times one needs to alternate concatenation and Boolean operations or, equivalently, counting how many alternations of existential and universal quantifiers are required in the axiomatization.

The problem proposed by Cohen and Brzozowski in 1971 was to compute the minimum number of such alternations required for a given star-free language. The case of one alternation was soon solved (Simon, 1975) and the so-called dot-depth hierarchy of regular languages thus defined was shortly after shown to be infinite (Brzozowski and Knast, 1978). But even to determine if it is decidable whether two alternations suffice remains an open problem. Straubing proposed in 1985 a conjecture for that case which has recently been shown to fail by O. Klíma and the speaker. Our method to construct a counter-example to the conjecture also leads to a new better upper bound for the class of dot-depth two languages. Through profinite methods and using Simon's Factorization Forest Theorem, we manage to prove that, like Straubing's upper bound, it is also decidable.

José Carlos Costa *CMAT and U. Minho*

MCCAMMOND'S SOLUTION OF THE OMEGA-WORD PROBLEM FOR FINITE APERIODIC SEMIGROUPS REVISITED (JOINT WORK WITH JORGE ALMEIDA AND MARC ZEITOUN)

An omega-term is an expression obtained from letters of an alphabet using the operations of concatenation and omega-power. These expressions can be naturally viewed as (implicit) operations on finite semigroups. The omega-word problem for a pseudovariety of semigroups V is the problem of determining whether two omega-terms represent the same operation on the elements of V.

This problem was solved by McCammond for the pseudovariety A of finite aperiodic semigroups, by defining a normal form for omega-terms and by showing that two omega-terms define the same operation over A if and only if they can be transformed into the same omega-term in normal form. He does that by appealing to his solution of the word problem for certain Burnside semigroups.

In this talk, we describe an alternative proof of McCammond's algorithm. Our approach consists in associating to each omega-term x and positive integer n a certain language L_n(x), and showing that the uniqueness of McCammond's normal forms is a consequence of two properties of such languages: for omega-terms x and y in normal form and a sufficiently large n: [1] L_n(x) and L_n(y) are star free languages; [2] if L_n(x) and L_n(y) are not disjoint then x=y.

Giovanni Pighizzini *Università degli Studi di Milano*

REMOVING NONDETERMINISM FROM TWO-WAY AUTOMATA

It is well-known that the capability of moving the input head in both directions does not increase the computational power of finite automata. In other words, two-way nondeterministic finite automata,(2NFAs), as well as two-way deterministic finite automata (2DFAs), characterize the class of regular languages. In 1978, Sakoda and Sipser posed the question of the cost, in term of states, of the simulation of 2NFAs by equivalent 2DFAs, and they conjecture that it is exponential. After many years, this problem is still open.

In the first part of the talk, we present this question and we discuss some related results. Subsequently, we consider the restriction to the case of unary automata, namely automata with a one letter input alphabet.

With a small increasing in the number of the states, each unary 2NFA can be converted into an equivalent 2NFA whose transitions satisfy some strong restrictions. This restricted form for 2NFAs was useful to obtain several interesting results:

(a) There exists a subexponential (but superpolynomial) simulation of unary 2NFAs by 2DFAs.

(b) Each unary 2NFA accepting a language L can be converted into a 2NFA accepting the complement of L, with only a polynomial increasing in the number of states.

(c) Ambiguity can be removed form unary 2NFAs, with only a polynomial increasing in the number of states.

(d) If L=NL (the classical logarithmic space classes) then each unary 2NFA can be converted into an equivalent 2DFA still keeping polynomial the number of states.

As a consequence of (d), proving that the simulation (a) is optimal, or proving a superpolynomial gap between unary 2NFAs and 2DFAs, would imply the separation of L from NL.

Rogério Reis *LIACC and U. Porto*

TWO FOR THE PRICE OF ONE

In this talk we analise the descriptional complexity of conversions between two regular language representations: finite automata nd regular expressions.

I) The partial derivative automaton (NFA_Pd) is usually smaller than other non-deterministic finite automata constructed from a regular expression, and it can be seen as a quotient of the Glushkov automaton (NFA_Pos). By estimating the number of regular expressions that have epsilon as a partial derivative, we compute a lower bound of the average number of mergings of states in NFA_Pos and describe its asymptotic behaviour. This depends on the alphabet size, k, and its limit, as k goes to infinity, is 1/2. The lower bound corresponds exactly to consider the NFA_Pd automaton for the marked version of the RE, i.e. where all its letters are made different. Experimental results suggest that the average number of states of this automaton, and of the NFA_Pos automaton for the unmarked RE, are very close to each other.

II) Algorithms for converting finite automata to regular expressions have an exponential blow-up in the worst-case. To overcome this, simple heuristic methods have been proposed. We analyse some of the heuristics presented in the literature and propose new ones.

Marco Almeida *LIACC*

INCREMENTAL DFA MINIMIZATION

The problem of finding the minimal DFA equivalent to a given automaton can be traced back to the 1950's with the works of Huffman and Moore. Having applications on compiler construction, pattern matching, hardware circuit minimisation, and XML processing to name a few, over the years several alternative (and increasingly efficient) algorithms were proposed.

We will present a new quadratic incremental DFA minimisation algorithm. Given an arbitrary DFA D as input, this algorithm may be halted at any time returning a partially minimised DFA that has no more states than D and recognises the same language. Whenever the minimisation process is interrupted, calling the incremental minimisation algorithm with the output of the halted process resumes the minimisation process. Being incremental also allows for the algorithm to be applied to an automaton D at the same time as D is being used to process a string for acceptance.

The algorithm uses a disjoint-set data structure to represent the DFA's states and Union-Find to mark pairs of equivalent states and to keep and update the equivalence classes. The pairs of states marked as distinguishable are stored in an auxiliary data structure in order to avoid repeated computations.

Unlike the more usual technique, which computes the equivalence classes of the set of states, this algorithm proceeds by testing the equivalence of pairs of states. The intermediate results are stored for the speedup of future computations in order to assure quadratic running time and memory usage.

Zur Izhakian *Bar-Ilan University*

SUPERTROPICAL ALGEBRAS

The objective of this talk is to introduce an algebraic structure rich enough to support algebraic formulations of properties of tropical algebraic geometry. Although this structure is a semiring, it permits a systematic development both of polynomials (and their roots) and of matrices, yielding direct analog to many results from commutative algebra and linear algebra, and thus is useful for semigroup representations.

In order to overcome the lack of additive inverses in idempotent semirings, we consider a semiring structure that has a distinguished "ghost ideal" taking the place of the zero element in many of the theorems. This leads to natural notions such as roots of polynomials, linear dependence of vectors, singularity and invertibility of matrices, rank of matrices, characteristic polynomial of matrices, eigenvalues, and eigenvectors.

Alfredo Costa *CMUC and U. Coimbra*

PROFINITE GROUPS ASSOCIATED TO SOFIC SHIFTS ARE FREE

Steinberg proved that the maximal subgroup of the minimal ideal of a free profinite semigroup is a free profinite group. He did so using sophisticated wreath product techniques.

This talk is about a joint work, authored by the speaker an Steinberg, where, with a refinement of these techniques, the following generalization is proved: the maximal subgroup of the free profinite semigroup associated by Almeida to an irreducible sofic shift is a free profinite group. A corresponding result is proved for certain relatively free profinite semigroups. Some other analogies between the minimal ideal of the free profinite semigroup and the -class associated to an irreducible sofic shift are established.

List of participants

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(8) Alfredo, Costa, *CMUC / Universidade de Coimbra*
(9) José Carlos, Costa, *CMAT / University of Minho*
(10) Manuel, Delgado, *CMUP / Universidade do Porto*
(11) Ana Paula, Escada, *Universidade de Coimbra*
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(14) Lucinda, Lima, *Universidade do Porto*
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