MINI COURSES

Quadratic (resp. odd-quadratic) Lie superalgebras
Saïd Benayadi

A quadratic (resp. odd-quadratic) Lie superalgebra is a Lie superalgebra with a non-degenerate supersymmetric, even (resp. odd), invariant bilinear form. Quadratic (resp. odd-quadratic) Lie superalgebras appear in particular in differential (super-)geometry and in physical models based on Lie superalgebras. The classification of quadratic (resp. odd-quadratic) classical simple Lie superalgebras had been obtained by V. Kac. But many solvable Lie superalgebras also belong to this class. In this mini course:

(1) We will construct some non-trivial examples of quadratic (resp. odd-quadratic) Lie superalgbras.
(2) Methods of double extension of quadratic (resp. odd-quadratic) Lie superalgebras will be developed.
(3) We will give some inductive descriptions of quadratic (resp. odd-quadratic) Lie superalgebras.
(4) We will introduce the quadratic dimension of a quadratic Lie superalgebra and we will establish some relations between this invariant and others invariants of quadratic Lie superalgebras.
(5) At the end, we will discuss some open problems on quadratic (resp. odd-quadratic) Lie superalgebras.

Gradings on simple Lie algebras
Alberto Elduque

A survey of some recent classification results for gradings over abelian groups on finite dimensional simple Lie algebras over algebraically closed fields will be given. This will require the classification of gradings on matrix algebras, on the algebra of octonions and on the Albert algebra (exceptional simple Jordan algebra).

Any grading of a finite dimensional algebra over an abelian group is equivalent to a representation of a diagonalizable affine group scheme on the affine group scheme of automorphisms of the algebra. This point of view, necessary to deal with the modular case, will be stressed throughout.

Kac-Moody algebras, vertex operators and applications
Vyacheslav Futorny

This mini-course will focus on vertex type constructions for certain classes of infinite-dimensional Lie algebras, including Affine Kac-Moody algebras and elliptic Affine algebras (the latter are particular cases of Krichever-Novikov algebras associated with elliptic curves). During the first lecture we will discuss some aspects of the representation theory of Kac-Moody algebras based on free field realizations of Affine Lie algebras and theory of vertex algebras. Vertex algebras have origin in string theory, they provide a mathematical foundation of 2-dimensional conformal field theory. Vertex algebras have numerous applications in many areas of mathematics, in particular they are ubiquitous in representation theory of infinite-dimensional Lie algebras. Some generalizations of vertex constructions for Affine Lie algebras will be considered. In the second lecture vertex realizations of Affine Lie algebras will be applied to the study of representations of the Lie algebra of vector fields on N-dimensional torus (these are joint results with Y. Billig). Finally, we will discuss free field realizations of elliptic Lie algebras which were recently obtained in a joint work with B. Cox and A. Bueno.

Differentiably simple (super)algebras
Alexander Pozhidaev

A superalgebra A is called differentiably simple if it lacks homogeneous ideals invariant under Der(A). In this talk we discuss the theory of differentiably simple (super)algebras and some its applications.