$\mathcal{VB}$–algebroids and representation theory of Lie algebroids

Alfonso Gracia–Saz
(joint with Rajan Mehta)

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- **Definitions:**
  - An A–*connection* on E is a smooth map ∇ such that:
    $$\begin{aligned}\nabla : \Gamma(A) \times \Gamma(E) &\to \Gamma(E)\\ (X \quad , \quad s) &\mapsto \nabla_X s\end{aligned}$$
    - $\nabla_X s$ is $\mathcal{C}^\infty(M)$–linear on X,
    - $\nabla_X(fs) = f\nabla_X s + \rho_A(X)(f)s$ for $f \in \mathcal{C}^\infty(M)$.

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    $$F_{X,Y} = \nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X,Y]}$$

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  - An A–*representation* on E, or a *flat* A–*connection* on E, is a connection with zero curvature.
- **Problem:** There is no adjoint representation.

Algebroid cohomology

- Let $A \to M$ be a LA. Define A–*forms*: $\Omega^p(A) = \Lambda^p\Gamma(A^\star)$.
- Define $d_A : \Omega^p(A) \to \Omega^{p+1}(A)$ by

$$d_A\omega(X_0,\ldots,X_p) = \sum_i (-1)^i \, \rho_A(X_i)\cdot\omega(\ldots,\widehat{X}_i,\ldots) \qquad \text{for } X_i \in \Gamma(A)$$

$$+\sum_{i<j}(-1)^{i+j}\,\omega([X_i,X_j],X_0,\ldots,\widehat{X}_i,\ldots,\widehat{X}_j,\ldots) \qquad \omega \in \Omega^p(A)$$

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  - It satisfies $d_A^2 = 0$,
  - and a Leibnitz rule; for $\omega_1, \omega_2 \in \Omega(A)$,

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- **Thm:** Let $A \to M$ be a VB.
  There is a one-to-one correspondence between:
  - Lie algebroid structures on $A \to M$, and
  - degree 1 operators d on $\Omega^\bullet(A)$ satisfying (1) and such that $d^2 = 0$.

Algebroid cohomology, with values in a vector bundle

- Let $A \to M$ be a LA. Let $E \to M$ be a VB.
  Define *E–valued A–forms*: $\Omega^p(A;E) := \Omega^p(A) \otimes_{\mathcal{C}^\infty(M)} \Gamma(E)$.
- Let ∇ be an A–connection on E.

  Consider it as a map $\nabla : \Omega^0(A;E) \to \Omega^1(A;E)$.

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  It can be extended to $D : \Omega^p(A;E) \to \Omega^{p+1}(A;E)$.
  satisfying a Leibnitz rule; for $\alpha \in \Omega(A)$, $\omega \in \Omega(A;E)$:

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- **Thm:** There is a one-to-one correspondence between:
  - A–connections ∇ on E, and
  - degree 1 operators D on $\Omega^\bullet(A;E)$ satisfying (2).

  Moreover, ∇ is flat iff $D^2 = 0$.

2.- Superrepresentations of Lie algebroids

- **Def:** Let $A \to M$ be a LA.
  Let $E \to M$ be a VB.
  An *A–representation* on E is
  a degree 1 operator D on $\Omega^\bullet(A;E)$ satisfying (2) and such that $D^2 = 0$.

$$\Omega^n(A;E) = \Omega^n(A) \otimes \Gamma(E)$$

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- **Def:** Let $A \to M$ be a LA.
  Let $\mathcal{E} \to M$ be a $\mathbb{Z}$–graded VB: $\mathcal{E} = \bigoplus_{n\in\mathbb{Z}} E^n$.
  An A–*superrepresentation* on $\mathcal{E}$ is
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- **Notes:**
  - When $\mathcal{E} = E^0$, we recover representations.
  - When $A = TM$, these are Quillen's flat superconnections.
  - Superrepresentations are called *representations up to homotopy* by Arias Abad and Crainic.

Example: case $\mathcal{E} = E^0 \oplus E^1$

A degree 1 operator on $\Omega(A;\mathcal{E})$ has four homogeneous components:

$$D = D^0 + D^1 + \partial + \Omega.$$

- $D^0 : \ \Omega^p(A, E^0) \to \Omega^{p+1}(A, E^0)$
- $D^1 : \ \Omega^p(A, E^1) \to \Omega^{p+1}(A, E^1)$
- $\partial : \ \Omega^p(A, E^0) \to \Omega^p(A, E^1)$
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Thm: An A–superrepresentation on $E^0 \oplus E^1$ is equivalent to:

- A–connections ∇^i on E^i, for $i = 0, 1$,
- a morphism of VBs $\partial : E^0 \to E^1$,
- a $\mathcal{C}^\infty(M)$–linear operator $\Omega \in \Lambda^2\Gamma(A^\star) \otimes \mathrm{Hom}(E^1, E^0)$

satisfying, for $X, Y \in \Gamma(A)$, and with F^i the curvature of ∇^i:

$$\partial \circ \nabla^0_X = \nabla^1_X \circ \partial \qquad F^0_{X,Y} = \Omega_{X,Y} \circ \partial$$
$$D^0\Omega + \Omega D^1 = 0 \qquad F^1_{X,Y} = \partial \circ \Omega_{X,Y}$$

The adjoint superrepresentation

Let $A \to M$ be a LA.
Choose a TM–connection on A

$$\tilde{\nabla} : \Gamma(TM) \times \Gamma(A) \to \Gamma(A)$$

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- $\nabla^1 : \Gamma(A) \times \Gamma(TM) \to \Gamma(TM)$ $\qquad \nabla^1_X \phi := [\rho_A(X), \phi]_{TM} + \rho_A(\tilde{\nabla}_\phi X)$
- $\partial : A \to TM$ $\qquad \partial = -\rho_A$
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$$\Omega_{X,Y}\phi = [\tilde{\nabla}_\phi X, Y]_A - [X, \tilde{\nabla}_\phi Y]_A - \tilde{\nabla}_\phi [X, Y]_A - \tilde{\nabla}_{\nabla^1_X \phi} Y + \tilde{\nabla}_{\nabla^1_Y \phi} X$$

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Problem: It depends on the choice of $\tilde{\nabla}$.

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- **Examples:** Let $A, E, C \to M$ be VBs.
Then the following are DVBs:

$$\begin{array}{ccc} TA & \longrightarrow & TM \\ \downarrow & & \downarrow \\ A & \longrightarrow & M \end{array} \qquad \begin{array}{ccc} T^\star A \cong T^\star A^\star & \longrightarrow & A^\star \\ \downarrow & & \downarrow \\ A & \longrightarrow & M \end{array} \qquad \begin{array}{ccc} A \times_M E \times_M C & \longrightarrow & E \\ \downarrow & & \downarrow \\ A & \longrightarrow & M \end{array}$$

Decompositions of double vector bundles

- **Lemma/Def:** Let D be a DVB.
  - Define $C := \ker q^D_A \cap \ker q^D_E$.
  - $C \to M$ is naturally a VB, called the *core*.

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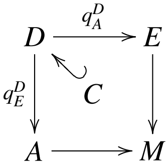

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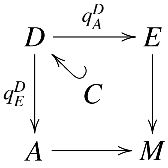

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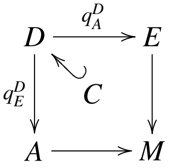
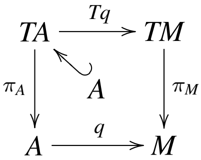


- **Lemma:** Every DVB has a decomposition, but not a canonical one.

- **Example:** Consider the DVB TA, for any VB $A \xrightarrow{q} M$:

There is a one-to-one correspondence between:

  - decompositions of TA, and
  - TM–connections on A.



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  - vertical arrows are VBs,
  - horizontal arrows are LAs,
  - plus compatibility conditions.

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- **Examples:** Let $E \to M$ be a VB, and $A \to M$ be a LA. Then the following are $\mathcal{VB}$–algebroids:

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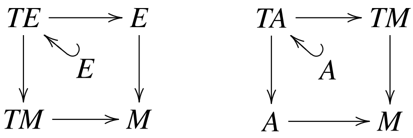

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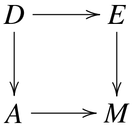

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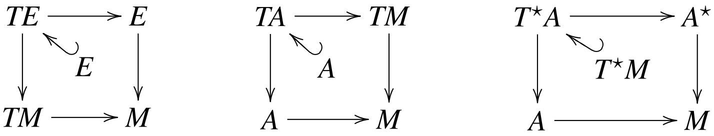
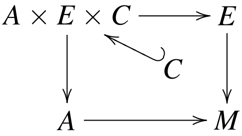


Main results

- **Thm:** [GS, MEHTA]
  Let $A \to M$ be a LA. Let $E, C \to M$ be VBs.
  There is a one-to-one correspondence between:
  - $\mathcal{VB}$–algebroid structures on the DVB $A \times E \times C$, and
  - A–superrepresentations on $C[0] \oplus E[1]$.



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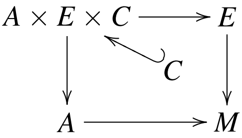

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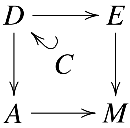

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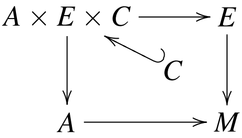

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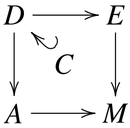

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  - ... equivalent to defining an A–superrepresentation on $C[0] \oplus E[1]$.

Conclusions

- **Analogy:**

$\mathcal{VB}$–algebroid	$\longleftrightarrow$	linear map
choice of decomposition	$\longleftrightarrow$	choice of basis
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- **The adjoint superrepresentation.**

$$\begin{array}{ccc} TA & \longrightarrow & TM \\ \downarrow & & \downarrow \\ A & \longrightarrow & M \end{array}$$

The adjoint superrepresentation of the LA $A \to M$ corresponds to the $\mathcal{VB}$–algebroid TA

After choosing a decomposition of TA
(or, equivalently, a TM–connection on A)
it is described by an A–superrepresentation on $A[0] \oplus TM[1]$.

Thanks.