Applications of Topology

IST - Lisbon 4th-7th February, 2015

XXIst Oporto Meeting on Geometry, Topology and Physics

Schedule

	February 4 wednesday	February 5 thursday	February 6 friday	February 7
9h-10h	D. Kozlov	M. Farber	M. Farber	D. Kozlov (9h30-10h30)
10h-10h50	M.Grant	A. Costa	L. Fajstrup	Z. Błaszczyk (10h30-10h50)
coffee break	Coffee + registration			
11h30-12h30	P. Sulkowski	U. Bauer	P. Sulkowski	U. Bauer
lunch				
14h-15h	U. Bauer	P. Sulkowski	D. Kozlov	M. Farber
15-15h50	T. Kahl	a) G. Minian b) B. Fasy	a) H. Colman b) J. Pita Costa	
16h30-17h30	a) M. Spera b) A. Salgueiro c) D. Govc	a) B. Gutiérrez b) M. Mamouni c) M. Bergomi	a) P. Lima Fortes b) M.J. Pereira c) J. Carrasquel	
17h50-19h00	a) S. Kallel b) P. Pavesik c) G. Lopes Cardoso	a) L. Velimirovic b) G. Petri c) D. Mondejar	a) A. Rieser b) A. Monod c) P. Lopes	
			conference dinner	

Abstracts

Speaker: Mark Grant, University of Aberdeen, UK

Title: Hopf invariants for sectional category with applications to Topological Robotics

Abstract: Topological complexity is a numerical homotopy invariant of spaces. It was defined by Farber as part of his topological study of the motion planning problem in Robotics. After reviewing the definition and basic properties, we will introduce refined homotopy-theoretic tools for the estimation of topological complexity, and more generally sectional category. These generalized Hopf invariants satisfy a sort of product formula, generalizing an observation originally due to N. Iwase. We will give applications to calculating the topological complexity of two-cell complexes and to the analogue for topological complexity of Ganea's conjecture on Lusternik-Schnirelmann category. This is joint work with Jesús González and Lucile Vandembroucq.

Speaker: Thomas Kahl, University of Minho, Portugal

Title: Directed algebraic topology of higher-dimensional automata

Abstract: Higher-dimensional automata constitute one of the most expressive models for concurrent systems. By definition, an HDA is a precubical set (i.e., a cubical set without degeneracies) with labels on edges. An important practical problem in concurrency theory is the fact that models of systems can easily become very large. This is called the state explosion problem. In this talk, I will discuss topological abstraction of higher-dimensional automata, i.e, the replacement of an HDA by a smaller one that is weakly equivalent from the point of view directed algebraic topology and models the same system.

Speaker: Mauro Spera, Università Cattolica del Sacro Cuore, Italy

Title: An application of Riemannian geometry to surveillance problems

Abstract: In surveillance applications, head and body orientation of people is of primary importance for assessing many behavioral traits. Unfortunately, in this context people are often encoded by a few, noisy pixels so that their characterization is difficult. We face this issue, proposing a computational framework which is based on an expressive descriptor, the covariance of features. Covariances have been employed for pedestrian detection purposes, actually a binary classification problem on Riemannian manifolds. In this paper, we show how to extend to the multiclassification case, presenting a novel descriptor, named weighted array of covariances, especially suited for dealing with tiny image representations. The extension requires a novel differential geometry approach in which covariances are projected on a unique tangent space where standard machine learning techniques can be applied. In particular, we adopt the Campbell-Baker- Hausdorff expansion as a means to approximate on th e tangent space the genuine (geodesic) distances on the manifold in a very efficient way. We test our methodology on multiple benchmark datasets, and also propose new testing sets, getting convincing results in all the cases. This is joint work with Diego Tosato, Marco Cristani and Vittorio Murino.

Speaker: António M. Salgueiro, University of Coimbra, Portugal

Title: A tool for the computation of persistent homology

Abstract: We describe and present a computer program devised to compute the persistent homology of a point set, and discuss some of the obstacles inherent to this computation.

Speaker: Dejan Govc, Institute of Math., Physics and Mechanics & Jožef Stefan Institute, Ljubljana, Slovenia

Title: New Results on the Unimodal p-Category

Abstract: The concept of unimodal category was introduced in 2007 by Baryshnikov and Ghrist as a topological abstraction of the statistical problem of representing a probability density function as a mixture of Gaussians. It can also be seen as a variation of the Lyusternik-Schnirelmann category. The problem of computing the unimodal category of a real function on a Euclidean space has been solved in dimension 1 and partially in dimension 2. In higher dimensions, not much is known. I will talk about new results on this and related questions. We have recently shown, for instance, that the monotonicity conjecture is, in general, false. To what extent does it hold?

Speaker: Sadok Kallel, American University of Sharjah, United Arab Emirates

Title: Barycenter spaces and their applications

Abstract: For a space X we associate the space of formal barycenters on n points of X. This is related to the join of X with itself n-times (symmetrized). This space appears in Topology in relation with stable splittings (Vassiliev). More importantly it appears in non-linear analysis in relation to the study of limiting Sobolev exponent problems such as the Yamabe and the scalar-curvature equations. Understanding the homology or homotopy type of barycenter spaces has important consequences in the field. We review the barycenter spaces construction and for special X we give a complete description of these spaces.

Speaker: Petar Pavešić, University of Ljubljana, Slovenia

Title: Topological complexity of kinematic maps

Abstract: In a mechanical device, like a robot arm, the forward kinematic map relates the configuration space of the joints to the working space of the device. A typical kinematic map is smooth but with certain singularities, and it admits only partial inverses. We are going to describe a general setting for the study of the motion planning problem in this context, and discuss the complexity of some simple joint configurations.

Speaker: Gabriel Lopes Cardoso, IST, Portugal

Title: Deformations of special geometry and the holomorphic anomaly equation

Abstract: The topological string captures certain superstring amplitudes which are also encoded in the underlying string effective action. However, unlike the topological string free energy, the effective action that comprises higher-order derivative couplings is not defined in terms of duality covariant variables. This puzzle is resolved in the context of real special geometry by introducing the so-called Hesse potential, which is defined in terms of duality covariant variables and is related by a Legendre transformation to the function that encodes the effective action. It is demonstrated that the Hesse potential contains a unique subsector that possesses all the characteristic properties of a topological string free energy. In particular, this subsector captures the holomorphic anomaly equation of perturbative type II topological string theory.

Speaker: Armindo Costa, Queen Mary, University of London, UK

Title: The fundamental group of a random clique complex

Abstract: Random objects often have desirable properties for which explicit examples are hard to construct. For example often random graphs are good expanders and have strong Ramsey properties. The most well-studied model of random graphs is the Erdos-Renyi model G(n,p). In the G(n,p) model one generates a random graph with n vertices by adding each possible edge with independent probability p. Properties of random graphs are often studied asymptotically, ie by having n tend to infinity and the probability parameter p depend on n. A pioneering result of Erdos and Renyi establishes the threshold, ie the critical p(n), for a random graph to be connected with probability tending to one.

In this talk we will study a model of random simplicial complexes introduced recently by M. Kahle. This model is known as the random clique complex model. Here a random complex is generated by first generating a random graph G in the Erdos-Renyi model and subsequently adding the faces spanned by complete subraphs (cliques) of G. Unlike in the graph setup, one can study several interesting topological

properties of random complexes. We will focus on properties of the fundamental group of a random clique complex. This is joint work with M. Farber and D. Horak.

Speaker: Gabriel Minian, University of Buenos Aires, Argentina

Title: A survey on the homotopy theory of finite topological spaces and applications

Abstract: I will show how finite topological spaces (i.e. topological spaces with a finite number of points) can be used to investigate classical (open) problems in topology. Finite spaces are closely related to finite posets, but the finite space point of view adds a new dimension to finite posets and allows the development of more appropriate techniques based on the combinatorics and the topology of these objects.

Speaker: Brittany T. Fasy, Tulane University, USA

Title: Road Network Comparison

Abstract: Road networks are always changing: new streets are built; accidents and floods close roads, etc. Detecting when and where a change has occurred is an important question. Surprisingly, only recently have distance measures between embedded graphs (representing road networks) been studied. In this presentation, we will discuss desirable properties of metrics between road networks, as well as present recent developments in this area, including a distance measure that uses a concept called local persistent homology.

Speaker: Bárbara Gutiérrez, CINVESTAV, Mexico

Title: The higher topological complexity of subcomplexes of products of spheres – and related polyhedral product spaces

Abstract: In this talk we will construct optimal "higher" motion planners for automated systems whose space of states are homotopy equivalent to a polyhedral product space $Z(K, \{(S^{k_i}, \star)\})$ with all of the k_i having the same parity. Our construction is shown to be optimal by explicit cohomology calculations. The higher topological complexity of other polyhedral product spaces is also determined.

Speaker: My Ismail Mamouni, CRMEF Rabat, Morocco

Title: Loop Topological Complexity

Abstract: The topological study of Robots motion planning algorithms emerged in the 2003-2004 with the works of M. Farber. His main tool was the concept of *Topological Complexity* denoted TC. Our aim in this talk is to introduce a similar one, the so-called *Loop Topological Complexity* denoted TC^{LP}. We prove that $\mathrm{TC} = \mathrm{TC}^{LP}$ and that it leads to a *loop motion product*. By the way, we also give some topological properties of the the set of motion planning algorithms. Many interpretations and open questions arise. This is joint work with Y. Derfoufi.

Speaker: Mattia G. Bergomi, IRCAM, France

Title: Dynamics in Modern Music Analysis

Abstract: From a dynamical point of view, isotropy is the main pathology of standard graph-based musical models such as the Tonnetz [Euler, 1739]. The main idea is to define a dissonance function for n-notes chords [Dillon, 2013] and use it to stretch the universal covering of the Tonnetz -seen as a simplicial complex embedded in $\mathbb{R}^3$ [Bigo et al., 2013]- to introduce preferential directions among notes. This kind of approach leads us to a characterization of musical objects (standard and altered

chords). The aim is to compute the persistent homology of the 3-dimensional cloud of points which is generated by the 0-skeleton of the simplicial complex we obtained reshaping the standard 2-dimensional Tonnetz. See [Verri et al., 1993, Edelsbrunner et al., 2002, Ghrist, 2008]. The integration of the set of persistent homological tools in music analysis and the study of the action of different filtrating functions on the complex could give a new point of view either on the Musical and Mathematical manifolds and in general on Global compositions, as they are described in [Mazzola et al., 2002, Chapter 13]. On the side of continuous models such as the chord spaces [Tymoczko, 2011], we suggest a braid-based interpretation of the orbifold structure of the model, in which the configuration space of $n \geq 2$ points in $\mathbb{R}^n/\Sigma_n$ (n-chords) has been analyzed in terms of the complexity of the braids group naturally associated to the orbifold [Birman, 1974]. Such an orbifold represents the voice leading space for a class of chord C, such that the number of notes of $c \in C$ is equal or less than n. These are the results of joint researches with Moreno Andreatta, Alessandro Portaluri, Riccardo Jadanza and Stefano Baldan.

Speaker: Ljubica Velimirović, University of Niš, Serbia

Title: On Shape at Infinitesimal Bending

Abstract: In this talk we will consider change of geometric magnitudes under infinitesimal bending. We will also analyse change of the Willmore energy and the mean curvature in such kind of deformations. The talk is based on the joint investigation with M. Cvetković, S. Minčić , M. Najdanović, M. Stanković and M. Zlatanović.

Speaker: Giovanni Petri, ISI Foundation, Italy

Title: Homological scaffolds of functional brain networks

Abstract: We study the characteristics of functional brain networks at the mesoscopic level from a novel perspective that highlights the role of inhomogeneities in the fabric of functional connections. We do this by focusing on the persistent homology associated with the weighted functional network. We leverage this topological information to define the homological scaffolds, designed to summarise compactly the homological features of the correlation network and simultaneously make their homological properties amenable to networks theoretical methods. As a proof of principle, we apply these tools to compare resting-state functional brain activity in 15 healthy volunteers after intravenous infusion of placebo and psilocybin?the main psychoactive component of magic mushrooms. We show that the homological structure of the brain's functional patterns undergoes a dramatic change post- psilocybin, characterized by the appearance of many transient structures of low stability and of a small n umber of persistent ones that are not observed in the case of placebo.

Speaker: Diego Mondéjar, Universidad Complutense de Madrid, Spain

Title: Shape Approximations of Compacta

Abstract: We will introduce the Theory of Shapes, a modification of K. Borsuk of homotopy theory to deal with compacta with bad local properties. Using this theory and given any compactum, we are able to construct a sequence of finite T_0 spaces such that its inverse limit represents the original space up to homotopy or shape type. As an application, this construction leads to a persistence module which can be used to infer some homology information of a point cloud.

Speaker: Lisbeth Fajstrup, Aalborg University, Denmark

Title: Directed Topology - dicoverings. Top versus dTop.

Abstract: Directed topology is a new mathematical area inspired by and with applications to concurrency. A topological space is directed by choosing a a subset of its paths, called the directed paths. Maps have to be continuous and respect this choice - directed paths are mapped to directed paths. In concurrency, algorithms and tools to investigate programs without loops are quite well developed in this geometric setting, and it is obvious from a topological viewpoint to try to get rid off loops by taking a (directed) covering. This talk will give examples and illustrate in what respect the usual definition actually works and where new ideas have to come in. In particular, the focus has to be on lifting properties, not the usual discrete fibration properties, which in the undirected setting imply lifting. The usual focus on connected spaces has to be modified, and again not in an obvious way. This tour through directed coverings will highlight many of the unusual and surprising features of directed topology.

Speaker: Hellen Colman, Wright College, Chicago, USA

Title: Synchronous Movement Planning

Abstract: Farber's topological complexity is a homotopy invariant which reflects the complexity of the problem of constructing a motion planning algorithm in the configuration space of a mechanical system. We introduce a groupoid invariant carrying an interpretation in terms of the motion planning problem for a robot when its configuration space exhibits symmetries. This number is an interesting invariant in itself to measure the complexity of motion planning algorithms in situations that might be modeled by a group action. In particular it provides a model for the planning of teams of robots moving synchronously on a physical space. This is joint work with Andres Angel.

Speaker: João Pita Costa, Institute Jozef Stefan, Slovenia

Title: Persistence on Sheaves over Lifetimes

Abstract: Persistent homology is a central tool in topological data analysis, which examines the structure of data through topological structure. The basic technique is extended in many different directions, permuting the encoding of topological features by barcodes and correspondent persistence diagrams. The set of points of all such diagrams determines a complete Heyting algebra that can explain aspects of the relations between correspondent persistence bars through the algebraic properties of its underlying lattice structure. A topos theoretic generalisation of the category of sets permits ideas as for sets varying according to time intervals. In general it provides tools for unification of techniques for mathematics. In this talk we shall look at the topos of sheaves over the algebra of lifetimes, discuss its construction and potential for a generalised simplicial homology over it. In particular we are interested in establishing a topos theoretic unifying theory for the various flavours of persistent homology that have emerged so far, providing a unification theory for the algebraic foundations of applied and computational algebraic topology.

Speaker: Paulino Lima Fortes, Universidade do Cabo Verde & CEMAT/IST, Cabo Verde

Title: On some topologic concepts in topologic vector spaces

Abstract: In this work, we introduce the concept of "inside" of a subset of a topologic vector space E, as well as some derived concepts and results. We use these

results in a topological characterization of subsets of E, complementary to the usual ones. We then finish the paper with some applications of the built framework to the study of the border of convex sets, pointing to applications in computational geometry.

Speaker: María José Pereira-Sáez, Universidade da Coruña, Spain

Title: LS category of Symplectic Grassmannians

Abstract: Given a topological space G, its LS category, cat$G$, is the least integrer $k \geq 0$ such that G can be covered by $k+1$ open sets contractible in G. The direct computation of this topological invariant is a hard job. But there are some bounds that make it easier. As the symplectic Grassmanians are 3-connected, the dimension upper bound and the lower bound of the cup-length lead us to the result $\mathrm{cat}G(k,n) = k(n-k)$. However, we have not any categorical covering. Our target is to give explicitly a covering by contractible open subsets. We will see how height functions and Morse theory can help us in this task.

Speaker: Jose G. Carrasquel-Vera, UCL Louvain-la-Neuve, Belgium

Title: On the sectional category of certain maps

Abstract: We give a simple characterisation of the sectional category of rational maps admitting a homotopy retraction which generalises the Félix-Halperin theorem for rational LS category. As a particular case, we prove a conjecture of Jessup-Murillo-Parent concerning rational topological complexity and generalise it to Rudyak's higher topological complexity.

Speaker: Antonio Rieser, Technion, Israel

Title: A Topological Approach to Spectral Clustering

Abstract: We present current work-in-progress using a new approach to data clustering through the analysis of the Laplace-Beltrami operator on a point cloud. In particular, given samples from a probability distribution on a submanifold M of Euclidena space, we combine a new family of approximations to the Laplace-Beltrami operator with a statistical model selection technique in order to find a 'topologically good' approximation to the number of connected components of M. We then use this to assign each point to one of the components, i.e. give a clustering of the data. We provide some experimental support for the conjecture that the algorithm produces the correct clustering with high probability as the number of samples increases, and, although our current theoretical methods are restricted to manifolds, we briefly explain why we believe the algorithm will work in more general settings as well.

Speaker: Anthea Monod, Duke University, USA

Title: Statistical Estimation of Random Field Thresholds Using Euler Characteristics

Abstract: We introduce Lipschitz-Killing curvature (LKC) regression, a new method to produce $(1-\alpha)$ thresholds for signal detection in random fields that does not require knowledge of the spatial correlation structure. The idea is to fit the observed empirical Euler characteristics to the Gaussian kinematic formula via generalized least squares, which quickly and easily provides statistical estimates of the LKCs – complex topological quantities that are otherwise extremely challenging to compute, both theoretically and numerically. With these estimates, we can then make

use of a powerful parametric approximation of Euler characteristics for Gaussian random fields to generate accurate $(1-\alpha)$ thresholds and p-values. Furthermore, LKC regression achieves large gains in speed without loss of accuracy over its main competitor, warping. We demonstrate our approach on an fMRI brain imaging data set. This is joint work with Robert Adler (Technion), Kevin Bartz (Renaissance Technologies), and Samuel Kou (Harvard).

Speaker: Pedro Lopes, IST, Portugal

Title: The delunification process and minimal diagrams

Abstract: A link diagram is said to be lune-free if, when viewed as a 4-regular plane graph it does not have multiple edges between any pair of nodes. We prove that any colored link diagram is equivalent to a colored lune-free diagram with the same number of colors. Thus any colored link diagram with a minimum number of colors (known as a minimal diagram) is equivalent to a colored lune-free diagram with that same number of colors. We call the passage from a link diagram to an equivalent lune-free diagram its delunification process. This is joint work with Slavik Jablan and Louis Kauffman.

Speaker: Zbigniew Błaszczyk, Adam Mickiewicz University, Poland

Title: On invariant topological complexity of smooth $\mathbb{Z}/p$-spheres

Abstract: We investigate invariant topological complexity of spheres endowed with non-free smooth $\mathbb{Z}/p$-actions. In particular, we show that invariant topological complexity distinguishes linear from smooth actions: a linear $\mathbb{Z}/p$-sphere S^n with a non-empty & connected fixed point set always has $2 \leq TC^{\mathbb{Z}/p}(S^n) \leq 3$, and this is usually not the case for non-linear spheres. We work with the notion of invariant topological complexity introduced by Lubawski and Marzantowicz, but similar results can be obtained for equivariant topological complexity of Colman and Grant.