



# Quadratic complexes, singular varieties and moduli

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**Abstract.** Let  $G$  be the Grassmannian of lines in  $\mathbb{P}^3$  embedded in  $\mathbb{P}^5$  as the Plücker quadric  $Q$ . The intersection of  $Q$  with a second hypersurface of degree  $d$  is what is called a complex of lines of degree  $d$ . When we consider the intersection of  $Q$  with a second quadratic hypersurface in  $\mathbb{P}^5$ ,  $P$ , we have a quadratic complex. Let  $X = Q \cap P$  be a quadratic complex that, in this talk, we assume to be non-singular, meaning  $X$  is non-singular.

The quadric  $Q$  contains a 3-dimensional family of planes parametrizing lines in  $\mathbb{P}^3$ , going through a point. These are known in the literature as  $\alpha$ -planes. An  $\alpha$ -plane,  $\alpha(p)$ , intersects the quadric  $P$  in a conic  $K_{\alpha(p)}$ . The singular surface  $S$  associated to the quadratic complex  $X$  is defined to be the  $p \in \mathbb{P}^3$  such that the plane  $\alpha(p)$  corresponding to  $p$  intersects the quadric  $P$  in a singular conic  $K_{\alpha(p)}$ .

$$S = \{p \in \mathbb{P}^3 \text{ such that } \text{rank}(K_{\alpha(p)}) \leq 2\}$$

All this is very classical and can be read for instance in the book by Griffiths & Harris, *Principles of Algebraic Geometry*. In a joint paper with H. Lange, (D. Avritzer e H. Lange, *Moduli spaces of quadratic complexes and their singular surfaces*, *Geom. Dedicata* V. **127** (2007) p. 177-179.), we studied the moduli spaces associated to this objects not only when  $X$  is non-singular but also in the singular case. It turns out that there is an equivariant map defined that associates to a quadratic line complex  $X$  its singular surface  $S$ . The inverse image of a given singular surface  $S$  is what is called the Klein variety.

In this seminar, I will explain these ideas and their relationship with the moduli space of vector bundles a result that goes back to a famous paper of Narasimhan & Ramanan and was proved independently by P. Newstead.

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15:30

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