A New Multiperiod Stage Definition for the Multistage Benders Decomposition Approach Applied to Hydrothermal Scheduling

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***Abstract*—Multistage Benders decomposition (MSBD), also known as dual dynamic programming, is a well-established technique to solve hydrothermal scheduling problems, especially for predominantly hydro systems. The MSBD methodology solves the problem by iterative forward and backward recursions, approximating the cost-to-go function for each stage by Benders cuts, as opposed to traditional dynamic programming approaches that discretize the state space at each time-step. The classical definition of the stages in the MSBD approach is to assign a stage for each time period. In this paper, we propose a new strategy to decompose the problem, where each stage comprises all variables and constraints of several time periods.**

Numerical results of the application of this strategy to the short-term hydrothermal scheduling problem confirm the advantages of this strategy in terms of CPU time, as compared to the classical stage definition approach. We show that there is an "optimal aggregation factor," which best balances the trade-off between solving a "larger number of shorter subproblems" and solving a "smaller number of larger subproblems." The primal and dual solutions related to different aggregation factors are also compared, and the stability of the results is confirmed. Extensions of the proposed strategy to stochastic problems are discussed.

***Index Terms*—Benders decomposition, dynamic programming, linear programming, power generation scheduling.**

NOMENCLATURE

$\alpha^T(\cdot)$	Future system costs as a piecewise linear function of the vector of storages in the reservoirs at the end of last time period T.
$\alpha^p(\cdot)$	Current piecewise linear approximation for the future cost function for each stage p in the Benders decomposition approach.
$c_i(\cdot)$	Piecewise linear generation cost function for thermal plant i.
D_i^t	Demand of area i at time period t.
Φ_i	Set of thermal plants belonging to area i.
Ψ_i	Set of hydro plants belonging to area i.
GT_i^t	Generation of thermal plant i at time period t.
GH_i^t	Generation of hydro plant i at time period t.
$\mathrm{HPF}_i(\cdot)$	Piecewise linear production function for hydro plant i.
I_i^t	Natural inflow to hydro plant i at time period t.
Int_{ij}^t	Energy interchange from area i to area j at time period t (negative values for flow from j to i).
M_i	Set of hydro plants immediately upstream plant i.
NA	Number of areas.
NH	Number of hydro plants.
NT	Number of thermal plants.
Ω_i	Set of areas directly connected to area i.
P	Number of stages in the MSBD approach.
Q_i^t	Turbined outflow of hydro plant i at time period t.
S_i^t	Spillage of hydro plant i at time period t.
T	Number of time periods (time-steps).
V_i^t	Storage of hydro plant i at the end of time period t.

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I. INTRODUCTION

THE operation planning of hydrothermal systems, usually called hydrothermal coordination (HTC), is a very complex optimization problem. Decisions to be made are coupled in time, as future reservoirs storages depend on the previous operation of the system. Generations of hydro and thermal plants must be coordinated, not only because of system constraints such as satisfaction of demand and reserve but also because of plant operation characteristics, such as hydro plants in cascade. In addition, uncertainties of both demand and hydrological conditions have to be managed.

The HTC problem is usually solved by decomposition of the original problem into long-, medium-, and short-term problems [1], [2], each one considering the appropriate aspects for its time-step and horizon of study. In general, uncertainties are modeled accurately in the long run, while system constraints are more detailed in the short-term horizon. Coordination among the models can be done either by setting targets [1] or by giving economic signs [2] to the downward models, in order to guarantee a proper system optimization.

One of the most used tools to solve the HTC problem, especially in predominantly hydro systems, is multistage Benders decomposition (MSBD), also known as dual dynamic programming approach [3]. The usual definition of stages in the MSBD approach is to assign a stage for each time-step.

The main contribution of this paper is to propose an alternative strategy to decompose the HTC problem when solving it by an MSBD approach. Instead of the usual decomposing of the problem into one stage for each time-step [4], in our approach each stage comprises variables and constraints of several time-steps. The advantage of this approach is that it allows exploring the best trade-off between solving a "larger number of shorter stages" and solving a "shorter number of larger stages." There is an optimal aggregation factor that yields the least CPU time to solve the overall problem, as can be seen in the numerical results presented to solve the deterministic short-term hydrothermal scheduling problem (STHTS) with 168 hourly time-steps.

This paper is organized as follows. In Section II, we formulate the STHTS problem considered in this paper, and discuss two approaches that can be applied to solve the general hydrothermal scheduling problem via Benders decomposition: a two stage and a multistage approach. In Section III, we describe the classical MSBD approach. In Section IV we propose a new definition of stages for the MSBD strategy in a deterministic framework. In Section V, we assess the performance of our approach to solve the STHTS problem and perform sensitivity analysis for some study cases based on the real Brazilian system. In Section VI, we discuss extensions of the proposed approach to the stochastic case. Finally, in Section VII we state the conclusions of this paper and discuss some future work.

II. Short-Term Hydrothermal Scheduling Problem

There are many different formulations for the short-term scheduling problem, depending on which variables are considered (e.g., whether unit commitment decisions are included or not) and which constraints are taken into account (e.g., type of modeling for the electrical network, set of thermal operating constraints, representation of the hydro plants production function, and so on). For a bibliographic survey, we refer to [5].

Because of the computational burden involved in solving a full 168-h scheduling problem taking into account accurately all system components and constraints, the short-term planning can be decomposed in two problems as follows:

- a one-week-ahead scheduling problem, with an hourly discretization and a continuous formulation [6], [7]. In this problem, the aim is to provide a proper transition between the midterm planning and the day-ahead scheduling. For example, in Brazil, there are water delay times of several days, which cannot be taken into account properly neither in the midterm planning (due to weekly time-steps) nor in the one-day-ahead problem (due to its short time horizon);
- a one-day-ahead unit commitment problem, where the status (on/off) of the units along the next day are determined, and the system is represented as accurately as possible. Ideally, this problem should have a nonlinear formulation (e.g., nonlinear transmission losses [8], an ac model for the electrical network [9], quadratic thermal generation costs [9], nonlinear hydro production function for the hydro plants [10], [11]), and take into account all hydro and thermal unit commitment related aspects (e.g., start-up costs [9], forbidden zones [10], [12] and ramping constraints [12]).

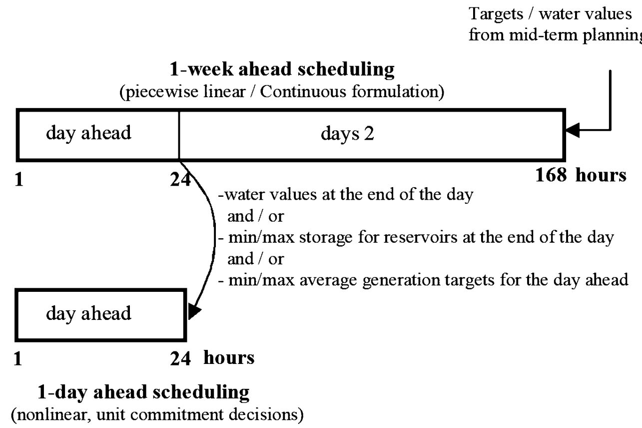


Fig. 1. Decomposition of the short-term planning into one-week-ahead and one-day-ahead scheduling problems.

The integration between the two models—which are intended to be run on a daily basis—is illustrated in Fig. 1.

In this paper, we consider the one-week-ahead scheduling problem with a continuous formulation and a cost minimization objective function. This problem will be denoted in the sequel as short-term hydrothermal scheduling problem (STHTS). We consider a "forecast" unit commitment for the next week, based upon maintenance scheduling and the load profile along the days. In order to avoid elimination of potential candidate solutions to the optimum, we only consider an "off" status for a unit for those hours of the day where the unit should be clearly shutdown, according to historical data and the information provided by the utilities.

Based on the results of this problem, water values and/or minimum/maximum generation targets are set to hydro and thermal units for the one-day-ahead scheduling problem, which will determine the status and the generation of the units for the next day.

We consider a multiarea system with NA areas, NT thermal plants, and NH hydro plants. The number of time periods is T. The problem is formulated as a linear program, as follows:

$$\text{minimize} \sum_t \sum_j c_j \left(GT_j^t\right) + \alpha^T(V^T) \tag{1}$$

s.t.

$$\sum_{j\in\Phi_k} \mathrm{GT}_j^t + \sum_{i\in\Psi_k} \mathrm{GH}_i^t + \sum_{l\in\Omega_k} \mathrm{Int}_{lk}^t = D_k^t \tag{2}$$

$$V_i^t + \left(Q_i^t + S_i^t\right) - \sum_{m\in M_i} \left(Q_m^t + S_m^t\right) = V_i^{t-1} + I_i^t \tag{3}$$

$$\mathrm{GH}_i^t = \mathrm{HPF}_i\left(V_i^{t-1}, V_i^t, Q_i^t, S_i^t\right) \tag{4}$$

$$\underline{\mathrm{GH}_i} \leq \mathrm{GH}_i^t \leq \overline{\mathrm{GH}_i} \quad (5a)$$
$$\underline{Q_i} \leq Q_i^t \leq \overline{Q_i} \quad (5b)$$
$$\underline{V_i} \leq V_i^t \leq \overline{V_i} \quad (5c)$$
$$0 \leq S_i^t \leq \overline{S_i} \quad (5d)$$
$$\underline{\mathrm{GT}_j} \leq \mathrm{GT}_j^t \leq \overline{\mathrm{GT}_j} \quad (5e)$$
$$\underline{\mathrm{Int}_{lk}^t} \leq \mathrm{Int}_{lk}^t \leq \overline{\mathrm{Int}_{lk}^t} \quad (5f)$$

where $i = 1, \ldots \mathrm{NH}, j = 1, \ldots \mathrm{NT}, k, l = 1, \ldots \mathrm{NA}, t = 1, \ldots T$.

Thermal generation costs are modeled by piecewise linear functions $c_j(\cdot)$. Hydro generation costs are evaluated implicitly by a multivariate future cost function $\alpha^T(\cdot)$. This function is computed by a midterm model and expresses the expected costs of thermal generation and energy deficit in the future as a function of the vector $V^T := \{V_i^T, i = 1, \ldots, \mathrm{NH}\}$ of storages in the reservoirs at the end of the time horizon [2].

Equation (2) is the load supply constraint for each area k, considering energy interchanges between areas. Equation (3) corresponds to the water balance equation for each hydro plant at each time period. Equation (4) describes the generation of the hydro plant as a piecewise linear function of the storage V in the reservoir, the turbined outflow Q, and the spillage S. A detailed description of this function is given in [13]. Finally, (5a)–(5f) state lower and upper bound for each set of variables.

Since the planning horizon is short (one week), we consider the problem as deterministic, which is a common assumption in short-term planning [5], [6], [8], [14]. Stochastic formulations of the short-term planning can be found in [9] (line contingencies), [15] (unit outages), and [16] (uncertainty on demand and streamflows). In Section VI we discuss extensions of the proposed approach to the stochastic case.

A. Benders Decomposition Applied to Hydrothermal Scheduling

Among several optimization techniques applied to hydrothermal scheduling, Benders decomposition [17] has presented very good results so far for large-scale systems [2]–[4], [8], [9], [18]–[20]. This technique has been applied based on two different approaches, as follows.

1) Two-Stage Approach: This strategy is usually applied when integer variables are introduced in the problem. The hierarchy of problems is defined by a high-level master problem that in general deals with integer variables, and several low-level subproblems. Examples of this approach can be found in [8], [9], and [18].

2) Multistage Approach: This strategy is referred to in this work as MSBD, and is also known as dual dynamic programming [3]. In this approach, applied previously to the long- and midterm operation planning [2], [3], [19], [20], a time decomposition is employed, and the subproblem for each stage t becomes a master problem for the subproblems from stages $t+1$ to T. This strategy requires the problem to be convex, as in (1)–(5f). In the applications mentioned above, a stochastic problem was addressed due to the much more extended planning horizon as compared to the one considered in this paper.

III. Classical Solution Strategy by MSBD

In this section we summarize the classical MSBD solution strategy. For more details we refer to [3] and [4].

Denoting by $\langle \cdot, \cdot \rangle$ the inner product between two vectors, the deterministic STHTS problem can be formulated in an abstract way as

$$\begin{aligned} &\min \sum_{t=1}^{T} \langle c^t, x^t \rangle + \alpha^T(x^T) \\ \text{s.t.} \quad &E^t x^{t-1} + A^t x^t \leq b^t, t = 1, \ldots T \\ &x^t \geq 0, \quad t = 1, \ldots T \end{aligned} \quad (6)$$

where x^t is the vector of decision variables for time period t (e.g., hydro generation, thermal generation, storage in the reservoirs), with associated cost vector c^t; E^t, A^t and b^t define the set of constraints for time period t, which may couple variables from different time periods. In this paper only a lag-one dependency among time periods is considered, in hydro balance constraints (3) and in the hydro plants production function (4). Both constraints link storage V^{t-1} at the end of time period $t-1$ to variables from time period t. In more complex formulations, higher lag dependencies may occur, for example, if we consider water delay times between cascaded reservoirs.

The MSBD methodology uses a time decomposition to obtain nested subproblems, each one representing a "stage" in a dynamic programming framework. The classical definition of stages in the MSBD approach [3]—which has been used by all works so far that applied such methodology (e.g., [2]–[4], [19], [20])—consists in one stage for each time period t. Therefore, the number of stages corresponds to the total number of periods T, and the subproblem $[t]$ of each stage t includes the variables and constraints related to period t, as follows:

$$\begin{aligned} &\min \langle c^t, x^t \rangle + \alpha^t(x^t) \\ \text{s.t.} \quad &A^t x^t \leq b^t - E^t \hat{x}^{t-1} \quad [t] \\ &x^t \geq 0 \end{aligned} \quad (7)$$

where $\hat{x}^{t-1}$ is the vector of state variables for subproblem $[t]$.

In the MSBD approach, contrary to Bellman's traditional dynamic programming approach [21], the future cost function for each stage t is approximated by applying Benders cuts, in an iterative procedure composed of successive forward and backward runs. The term $\alpha^t(x^t)$ in the objective function of (7) is the approximation of the future cost function for time period t at the current iteration.

Each iteration of the MSBD consists of:

- a forward simulation, from stages 1 to T, where values of state variables for each stage are obtained from the solution $\hat{x}^{t-1}$ of the previous stage;
- a backward recursion from stage $T-1$ to 1, using as state variables for each stage t the same solution $\hat{x}^{t-1}$ obtained at the latest forward run. During this process, a new Benders cut to refine the future cost function for each stage $t-1$

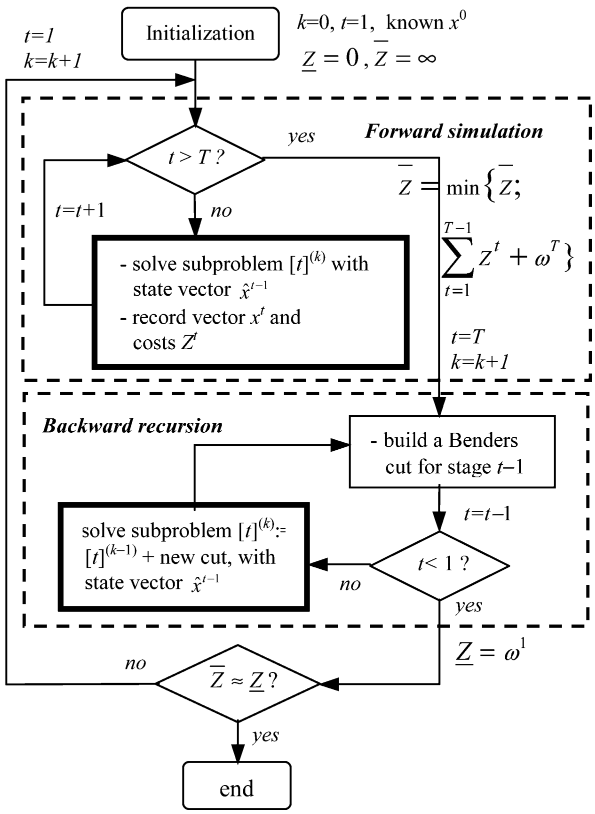


Fig. 2. Flow chart of the MSBD approach to solve the deterministic STHTS problem.

is obtained from the solution obtained for time period t, as follows:

$$\alpha^{t-1} \geq \omega^{t*} + \left\langle \frac{\partial \omega^t}{\partial x^{t-1}}(\hat{x}^{t-1}), x^{t-1} - \hat{x}^{t-1} \right\rangle \tag{8}$$

where $\omega^{t*} = \langle c^t, x^{t*} \rangle + \alpha^t(x^{t*})$ is the optimal value for subproblem t at current iteration, with optimal solution x^{t*}; the first term defining ω^{t*} corresponds to Z^t in Fig. 2 and is the cost associated to variables of the current time period; the second term in ω^{t*} is the future cost of stage t; $\hat{x}^{t-1}$ is the vector of state variables for stage t at current iteration (obtained from the solution of stage $t-1$ in the forward simulation), and $(\partial \omega^t)/(\partial x^{t-1})(\cdot)$ is the vector of partial derivatives of ω^t with respect to the state variables of subproblem t.

Equation (8) defines a multivariate linear cut to the future cost function of stage $t-1$, as a function of state variables x^{t-1} to subproblem $[t]$. For the STHTS problem formulated in this paper, the state variables $\hat{x}^{t-1}$ are the storages V^t in the reservoirs at the end of time period $t-1$.

Denoting by $[t]^{(k)}$ the subproblem for time period t at iteration k, the flow chart of Fig. 2 illustrates the MSBD strategy for this deterministic case.

In each forward simulation, the upper bound $\overline{Z}$ for the optimal solution Z^* of the problem is updated. The total cost of the subproblem for the first stage at the end of each backward recursion gives an updated lower bound $\underline{Z}$. In this deterministic version of MSBD, the solution procedure stops when the relative difference $(\overline{Z} - \underline{Z})/\underline{Z}$ is within a certain tolerance ε. In the stochastic version [3], a sample of scenarios can be generated by a Monte Carlo simulation, and the procedure stops when the difference between $\underline{Z}$ and $\overline{Z}$ falls within a certain confidence interval.

IV. Proposed Multiperiod Stage Definition for the MSBD Approach

In the classical MSBD approach presented in the previous section, the number of stages grows linearly with the size of the planning horizon. An example is the 168-stage problem for the hourly short-term scheduling problem within a one-week horizon. Numerical experience by the authors for solving this problem has shown that a very large number of stages leads to too many MSBD iterations and may slow down the convergence process. Such difficulty takes place especially when constraints coupling several time periods are introduced in the problem—as, for example, extended water delay times between hydro plants in cascade and total weekly or monthly generation targets to hydro/thermal plants. Another negative effect of a large number of iterations for convergence is that accumulation of Benders cuts—some of them very similar—may cause numerical difficulties in solving subproblem $[t]$ for each stage.

In order to decrease the number of stages of the problem—and, as a consequence, speed up the convergence of the MSBD and avoid numerical difficulties—we propose a new decomposition scheme, where each stage comprises variables and constraints for several time periods.

A. Stage Definition

The set $\{1, \ldots, T\}$ of time periods is partitioned into P stages. The first and last time periods of stage p are denoted by $\underline{t_p}$ and $\overline{t_p}$, respectively. The boundary conditions are $\underline{t_1} = 1$ and $\overline{t_P} = T$, and the general rule is $\underline{t_{p+1}} = \overline{t_p} + 1$. This approach is illustrated in Fig. 3, for an example with $T = 12$ and four time periods per stage. In this case, $P = 3; \underline{t_1} = 1; \overline{t_1} = 4; \underline{t_2} = 5; \overline{t_2} = 8; \underline{t_3} = 9; \overline{t_3} = 12$.

In this new decomposition approach, we denote the average number K of time periods per stage $(K = T/P)$ as "aggregation factor."

The subproblem for each stage p is denoted by $[p]$ and its formulation includes the variable and constraints related to all time periods from $\underline{t_p}$ to $\overline{t_p}$. The formulation for each subproblem $[p]$ in the lag-one dependency problem (1) is

$$\begin{aligned} \min \; & \left[\sum_{t=\underline{t_p}}^{\overline{t_p}} \langle c^t, x^t \rangle \right] + \alpha^p(x^{\overline{t_p}}) \\ \text{s.t.} \; & A^t x^t \leq b^t - E^t \hat{x}^{t-1}, \quad t = \underline{t_p} \quad [p] \\ & A^t x^t + E^t x^{t-1} \leq b^t, \quad t = \underline{t_p} + 1, \ldots, \overline{t_p} \\ & x^t \geq 0, \quad t = \underline{t_p}, \ldots, \overline{t_p}. \end{aligned}$$

We emphasize that the proposed strategy does not lead to a loss in accuracy in the time discretization of the problem, which is still subdivided into T hourly time periods. The difference between the proposed approach and the classical stage definition approach is that, in our approach, constraints related to consecutive time periods may be gathered together in the same stage,

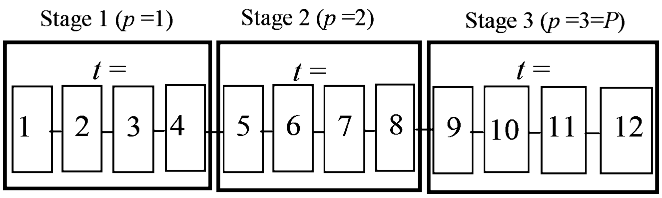


Fig. 3. Example of a multiperiod stage definition for the MSBD approach.

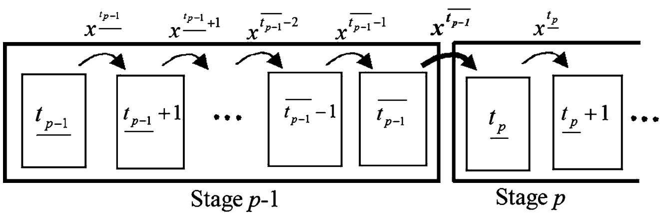


Fig. 4. State variables of the MSBD approach with the new stage definition, for a problem with lag-one dependency among time periods.

which means, in the same linear programming subproblem. In the same way, nothing is lost in terms of system representation and constraints modeling, since in the proposed approach we still have, for example, T hydro balance (3) and T hydro generation functions (4), one for each time-step.

B. Solution Strategy by MSBD

The solution strategy by MSBD with this new structure of stages is similar to the procedure shown in Fig. 2. The two main differences between the new approach and the classical approach are:

- in this new approach, the vector of state variables for stage p is composed only by decision variables for time period $\overline{t_{p-1}}$. All constraints that include variables $x^{\underline{t_{p-1}}}, x^{\underline{t_{p-1}}+1}, \ldots, x^{\overline{t_{p-1}}-1}$ appear in the same linear program of stage $p-1$, as illustrated in Fig. 4. For this reason, such variables are no longer state variables for the next stage in the iterative procedure shown in Fig. 2. As a result, the number of state variables of the MSBD approach is dramatically reduced with this new definition of stages;
- in the MSBD strategy, a future cost function (FCF) is obtained at the end of each stage. So, in this new approach, we obtain a FCF only for those time periods located at the end of each stage, rather than a FCF for each time period.

The Benders cuts for the FCF of stage $p-1$ become a function only of variables $\overline{t_{P-1}}$:

$$\alpha^{p-1} \geq \omega^{p*} + \left\langle \frac{\partial \omega^p}{\partial x^{\overline{t^{p-1}}}}(\hat{x}^{\overline{t^{p-1}}}), x^{\overline{t^{p-1}}} - \hat{x}^{\overline{t^{p-1}}} \right\rangle$$

where the optimal value and derivatives for each subproblem are indexed by the stage p rather than by the time period t as in (6).

C. Discussion

A careful implementation of the multiperiod MSBD approach proposed in this paper must take into account the following aspects:

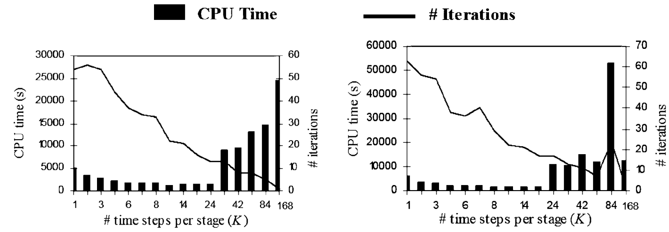


Fig. 5. Number of iterations and CPU time. Cases A (left) and B (right).

- the time periods for which one would like to have an FCF must match the end of some stage. For example, in the integrated short-term planning shown in Fig. 1, it is necessary to have an FCF at the end of the first day, to be used as input data to perform the one-day-ahead scheduling planning, where unit commitment decisions will be determined;
- the increase in the aggregation factor K (the number of time periods per stage) yields a reduction in the number of iterations for convergence of the MSBD approach (see results in Figs. 5 and 6). However, the CPU time to solve each subproblem at each iteration also increases with K, as the linear program of each stage becomes larger. So, the definition of K must take into account the performance of the solver employed to solve these linear programs;
- we have considered so far a uniform value of K along the entire planning horizon, which means that all stages comprise an equal number of time periods. However, experience in the implementation of the MSBD approach with the classical stage definition shows that subproblems related to peak hours take a longer time to be solved. This occurs, for example, when security constraints are introduced in the problem [22], as the number of binding constraints is greater in these peak-hour subproblems. Thus, in order to improve the performance of our proposed approach, a nonuniform definition of stages may be employed, where a larger number of time periods per stage can be used at lower load level hours.

V. Numerical Results

We considered the STHTS problem (1)–(5f) for a one-week horizon, discretized in hourly time periods ($T = 168$). This choice of T is very suitable for the sensitivity study to be performed, as 168 is a factor of 1, 2, 3, 4, 6, 8, 12, 14, 21, 24, 28, 42, 56, 84, and 168.

The study case is based on real data for the operation of the Brazilian system performed by the Brazilian Independent System Operator in November 2006. The system comprises 111 hydro plants (49 reservoirs and 62 run-off-the-river plants) and 49 thermal plants. System data can be obtained at http://www.ons.org.br. We used the IBM OSL package [23] to solve the linear programs of each stage. Tests were performed on a Pentium 4–3.00 GHz/504 MB RAM computer.

A. Consistency Analysis

In a first step, the aim is to perform a consistency analysis of the MSBD approach with the new stage definition proposed

TABLE I
CONSISTENCY ANALYSIS OF THE MSBD APPROACH WITH A NEW STAGE DEFINITION

k	$\underline{Z}$	$\overline{Z}$	CPU time (hh:mm:ss)
1	47,207,316.6	47,207,316.6	03:03:54
2	47,207,316.6	47,207,316.6	01:29:51
3	47,207,316.6	47,207,316.6	00:56:35
4	47,207,316.6	47,207,316.6	00:48:42
6	47,207,316.6	47,207,316.6	00:33:15
7	47,207,316.6	47,207,316.6	00:30:20
8	47,207,316.6	47,207,316.6	00:26:53
12	47,207,316.6	47,207,316.6	00:25:44
14	47,207,316.6	47,207,316.6	00:20:16
21	47,207,316.6	47,207,316.6	00:19:47
24	47,207,316.6	47,207,316.6	00:20:59
28	47,207,316.6	47,207,316.6	00:21:00
42	47,207,31**9.9**	47,207,31**7.5**	02:12:46
56	47,207,316.6	47,207,316.6	05:20:02
84	47,207,316.**7**	47,207,316.6	06:39:34
168	47,207,316.6		01:59:39

TABLE II
ADDITIONAL STUDY CASES FOR THE TRADE-OFF ANALYSIS

Case	Additional constraints introduced
A	Additional generation limits constraints for thermal and hydro plants
B	Case A + minimum outflow constraints for hydro plants
C	Case A + maximum storage constraints for flood control purposes
D	All constraints for cases A, B and C

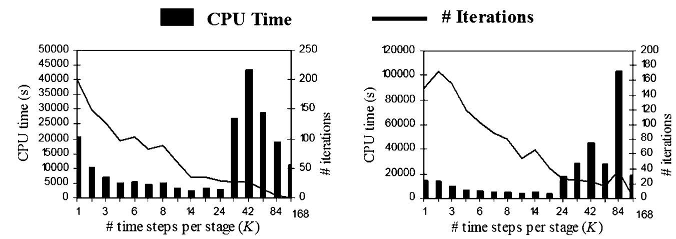


Fig. 6. Number of iterations and CPU time. Cases C (left) and D (right).

in this paper. For this reason, an extremely small tolerance for convergence of the MSBD iterative procedure of Fig. 2 was considered ($\varepsilon = 10^{-8}\%$).

Performance results for each value of K are shown in Table I, which lists the lower $(\underline{Z})$ and upper bounds $(\overline{Z})$ obtained at the end of the process, as well as the CPU time to achieve the stopping criteria. As the STHTS problem to be solved is the same for all cases, the optimal value does not vary with the aggregation factor K adopted. This was confirmed by our results, up to nine significant digits. The only exceptions—highlighted in bold font—were for $K = 42$ and $k = 84$. In these cases, numerical difficulties to solve some subproblems were reported by the OSL solver. We note that, for $K = 168$, the overall problem is solved with no decomposition (all 168 time periods are gathered in a same linear program), and in this case, the Benders decomposition procedure is not employed.

CPU times decrease fast as the value of K is increased, up to the value $K = 21$, which is the one that yielded the best performance. The CPU time for $K = 21$ is about one-ninth of the CPU time to solve the problem by the classical stage definition when Benders decomposition is employed $(K = 1)$ and about one sixth of the time to solve the problem as a single linear program $(K = 168)$. For values of K greater or equal than 42, CPU times increase in a very fast rate. It is interesting to note that for $K = 168$, the CPU time is smaller than for $42 \leq k < 168$. This suggests that, when the linear subproblems become very large, it may be more advantageous to solve the STHTS problem as a unique linear program, instead of applying the MSBD approach with a small number of stages.

B. Trade-Off Analysis

As discussed earlier, the use of an aggregation factor K in the definition of stages introduces a trade-off between the reduction in the number of iterations of the MSBD approach and the increase in the CPU time to solve each linear subproblem. To better assess this trade-off, we considered four additional study cases, labeled from A to D, where additional operation constraints were introduced in the STHTS problem, as described in Table II. In these study cases, the tolerance for convergence was increased to $10^{-4}\%$.

We show in Figs. 5 and 6 the number of iterations and CPU times for cases A to D.

The general behavior was similar to all study cases: an almost monotonic reduction in the number of iterations as the value of K increases, and an asymmetric "U" shaped behavior for the CPU time, with sudden increases when K is near to 24 or 28. The CPU time for the strategy with no decomposition $(K = 168)$ was smaller than those for large values of K, except for case A.

Comparing the results of these study cases to the results of the previous section, we conclude that, as the complexity of the problem increases (due to the inclusion of several additional operation constraints), the optimal aggregation factor (which we will denote as K^*) decreases. From cases A to D, such value K^* was around 12 and 14. We conclude that a large number of variables and constraints for each time period discourages larger aggregations, as the linear program of each stage becomes very large.

Table III reports, for each study case, the CPU times obtained with $K = 168, K = 1, K = K^*$ and with the value of K that yielded the worst results.

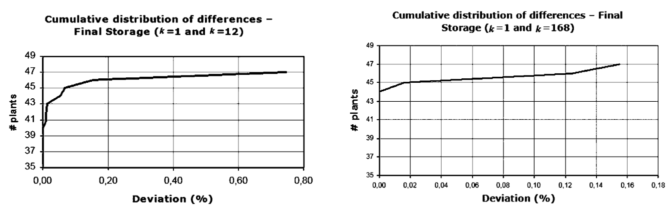
The average reduction in CPU time of the best aggregation strategy ranged from 2 to 8 times [if compared to the classical stage definition $(K = 1)$] and from 4 to 19 times [if compared to the single linear program approach $(K = 168)$].

C. Stability of Primal and Dual Solutions

In this section, we compare the solutions obtained with different aggregation factors for the problem in Section V-A. Three aggregation factors were considered: the classical approach $(K = 1)$, the "non-decomposition" approach $(K = 168)$, and the approach that yielded the least CPU time. We compared the

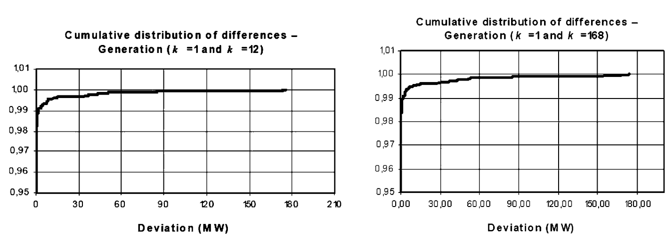
TABLE III
CPU TIMES FOR THE BEST, WORST, AND $k = 168$ PERFORMANCES

Case	Single Linear Program		Classical stage definition for MSBD		Worst performance		Best performance	
	K	CPU time (s)	K	CPU time (s)	K	CPU time (s)	K	CPU time (s)
A	168	24,600	1	4,800	168	24,600	12	1,260
B	168	12,436	1	6,127	84	50,400	12	1,320
C	168	11,109	1	20,866	42	43,200	14	2,400
D	168	19,111	1	13,610	42	45,000	12	3,420



(a) (b)

Fig. 7. Cumulative distribution of differences in final storage of the reservoirs, for the base-case with variants $K = 1, K = 12$, and $K = 168$.



(a) (b)

Fig. 8. Cumulative distribution of differences in hydro generation, for the base-case with variants $K = 1, K = 12$, and $K = 168$.

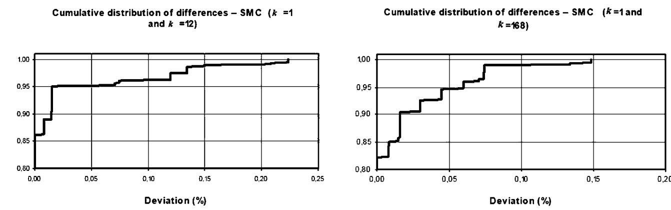
values of: storage in the reservoirs at the end of the week; hydro generations along the week; and system marginal costs along the week. The compared values were rounded to a precision of 0.01 (in % or MW, depending on the case), which is the one used by the Brazilian Independent System Operator to assess the results of the model.

1) Comparison of Storages: Fig. 7 shows the cumulative distribution of percentage differences in the storages of all 49 reservoirs in the configuration, between the $K = 1$ and $K = 12$ variants (on the left), and between the two extreme alternatives $K = 1$ and $K = 168$ (on the right). The remaining hydro plants operate as run-off-the river units.

The difference was zero (within the considered precision) for 40 plants in Fig. 7(a) and for 44 plants in Fig. 7(b). For the other reservoirs, differences were always lower than 0.20%, except for one reservoir, where the difference was near 0.80%.

2) Comparison of Hydro Generations: The same type of comparison is shown in Fig. 8 for the hydro generation values. The cumulative distribution was computed considering all $\mathrm{NH} \times T$ generation values that comprise all plants and time periods.

In both graphs, 98% of the differences were null. The largest difference was approximately 170 MW, in a hydro plant whose capacity is 1200 MW.



(a) (b)

Fig. 9. Cumulative distribution of differences in system marginal costs, for the base-case with variants $K = 1, K = 12$, and $K = 168$.

3) Comparison of System Marginal Costs: Finally, in Fig. 9, we show the cumulative distributions of the differences in the values of marginal costs, considering all system areas and time periods.

In this case, differences are higher: 5% are larger than 0.08% in Fig. 9(a) and 5% are larger than 0.06% in Fig. 9(b). Such behavior indicates that in the Benders decomposition approach, the stability of primal variables is much higher than for dual variables. However, differences are not so high, since the largest ones are lower than 0.24%.

These comparative results show that the final solution obtained by the DDP approach is not so much dependent on the aggregation factor applied in the proposed approach to solve the problem. In the case the optimality criteria for convergence of the DDP approach is decreased to values lower than $10^{-4}\%$, we except the differences to be even lower.

D. Analysis of "Bang-Bang" Behavior

As described in Section II, the proposed model for the STHTS problem is intended to be run previously to the model for the one-day-ahead scheduling problem for each day. In order to provide a proper integration between these two models, it is important to obtain for the STHTS problem a solution that is not so far from a feasible scheduling for the one-day-ahead scheduling problem, regarding the unit commitment constraints. This way, we could consider that the water values and/or operation targets set by the STHTS problem to the one-day-ahead scheduling problem are realistic.

In particular, we are interested in analyzing the bang-bang behavior of the generation of the hydro plants, which may occur when the STHTS problem is formulated by linear programming. We show in Figs. 10 and 11 the generation along the week for Itaipu and Xingó hydro plants, two plants that are crucial to follow the daily load curve. Itaipu is the largest hydro plant of the system (12 600 MW of installed capacity), located in the largest Brazilian river basin (Paraná) in the south part of the country. Xingó is located in the São Francisco river basin and is important to follow the load curve for the Northeast region of the country. The commitment status assumed for all units of these plants during the whole week was "on."

The hourly variations in the generation of both hydro plants are quite acceptable as a reference for the unit commitment problem. For Itaipu hydro plant, the unit commitment problem would probably handle the decrease in the generation in the first day by shutting down some of their units in the morning.

The average hourly percentage variation in the generation values per hydro plant (considering in this computation all hydro

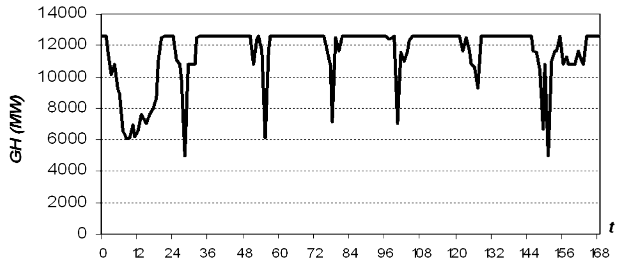


Fig. 10. Generation of Itaipu hydro plant along the week.

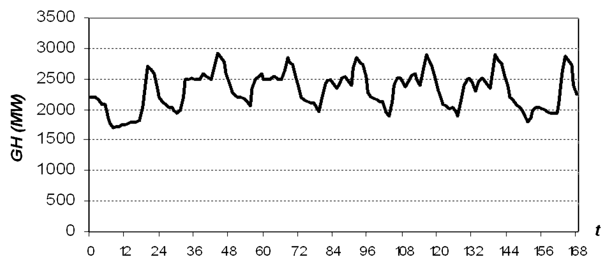


Fig. 11. Generation of Xingó hydro plant along the week.

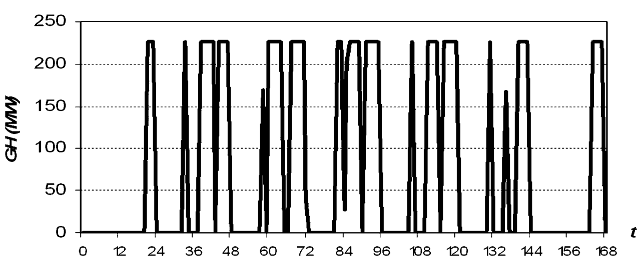


Fig. 12. Generation of Passo Fundo hydro plant along the week.

plants of the system) was about 15%, which is also a reasonable value. We note that the hydro production function is not modeled by a simple linear function, but rather by a detailed piecewise linear function [13] that becomes closer to the actual nonlinear function. In this sense, the bang-bang behavior is highly reduced, as suggested by the sensitivity analysis presented in [24] when the number of breakpoints is increased.

For the two plants presented above, the generation profile could be obtained by keeping all units on during the whole week and adjusting their generation levels. We show in Fig. 12 a different pattern that occurred for Passo Fundo hydro plant, which has only two units and a total capacity of 226 MW.

The generation of this plant was null for some time periods and there was a bang-bang behavior between minimum and maximum generation at some hours. Even though the STHTS already indicates commitment/decommitment operations for the units of this plant, the final decision is left for the unit commitment problem, since it would consider the trade-off between the benefits of the generation of this plant and the cost associated with several startups during the day.

We note that such behavior was found for very few—and small—hydro plants of the system.

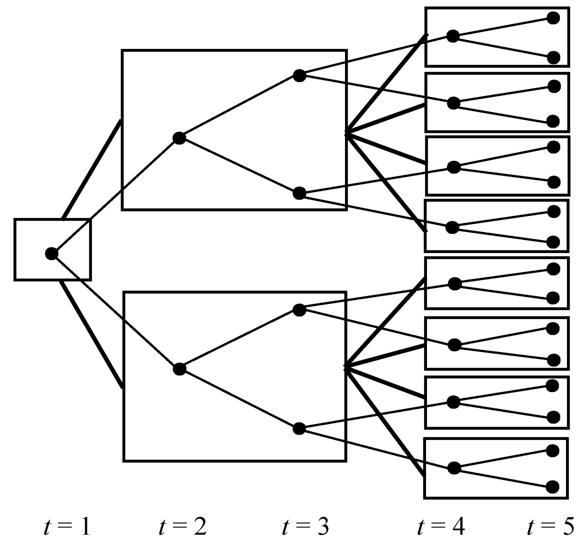


Fig. 13. New stage definition for the stochastic MSBD approach with a scenario tree representation.

VI. Extensions to the Stochastic Case

In this section, we discuss extensions of the proposed aggregation approach to stochastic problems. We consider two basic approaches to model the uncertain variables.

A. Scenario Tree Representation

In this approach, it is assumed that all possible realizations of the stochastic variables are represented by a scenario tree [16]. We propose for this case to perform a "scenario/period" aggregation approach. As a result, we would obtain a new outer tree whose nodes are sub-trees of the original tree, as illustrated in Fig. 13.

B. Monte Carlo Simulation

In this case, the optimization problem can be represented by the scheme shown in Fig. 14, where a sample of parallel scenarios is generated. In the backward run, we solve n subproblems at each node, corresponding to all different realizations of the stochastic variables considered at each time period. Details of this algorithm are given in [3].

Two alternatives are considered for the extension of the aggregation approach to this stochastic problem. The first alternative—indicated by letter "A" in Fig. 14—is to aggregate subproblems for each scenario. However, in this case, we could have difficulties in representing the stochastic process, because in a hazard-decision approach, it is assumed that at the beginning of each time period, there is a perfect knowledge of the uncertainty in this period, but not in the subsequent periods of the same stage. For this reason, we have to represent combinations of possible realizations of uncertain variables for all time periods within the stage. The second alternative—indicated by letter "B" in Fig. 14—is to aggregate different subproblems for each time period. In this case, we would solve simultaneously several independent subproblems, one for each scenario.

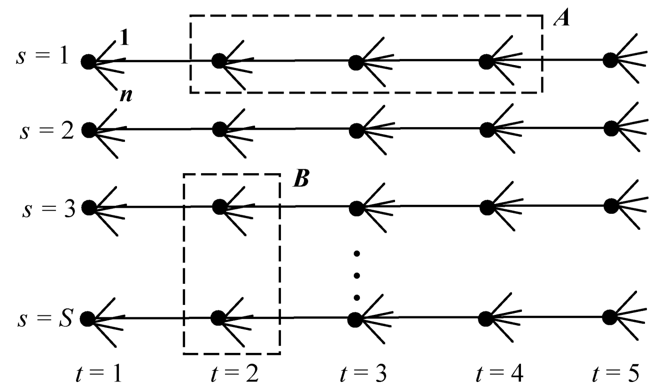


Fig. 14. Representation of the optimization problem when the stochastic process is represented by a Monte Carlo approach.

C. Discussion

The consideration of uncertainties is much more important in the midterm and long-term planning, since the impact of inflow and load forecast errors are smaller in the short-term planning.

For example, in Brazil, it is very difficult to foresee the water inflows seven days ahead. However, since the dispatch problem presented in this paper is intended to be solved by the independent system operator (ISO) on a daily basis, only the operation of the first day will be used by the ISO in the system operation. The operation of the next days will be determined again when the problem is run for the following days of the week, with new inflow forecasts. In this sense, the ISO will always use for the day-ahead the results obtained considering the one-day-ahead forecast for this day, where errors are much smaller.

The drawback of the aforementioned procedure is that some volatility could occur if the one-day-ahead forecast differs significantly from the $2, 3, \ldots 7$-day-ahead forecasts for a specific day. For this reason, the best approach would be to solve a stochastic version of the problem. However, due to the high CPU time already involved in solving the deterministic version of the problem (almost one hour for case D), we left this alternative as a future research.

VII. Conclusion

This paper proposed a new decomposition of the deterministic short-term hydrothermal scheduling problem, to be employed in an MSBD framework. In the classical MSBD approach, each stage is related to a single time period. By contrast, in our approach, each stage comprises all variables and constraints for several time periods. The distribution of time periods along the stages is defined by an aggregation factor K. We analyzed the trade-off between the two main effects of the increase in the value of K: the reduction in the number of iterations for convergence of the MSBD approach and the increase in the CPU time to solve the linear program for each stage.

Results confirmed the advantages of the new decomposition in terms of CPU time. In our studies, reductions in the overall CPU time to solve the problem depended on the case, with a minimum reduction of two times as compared to the classical MSBD approach and four times as compared to the single linear program approach. Sensitivity analysis on the value of K showed that there is an "optimal aggregation factor" K^* that yields the least CPU time. We also showed that such value K^* depends on the size and complexity of the problem: larger and more complex problems result in lower values of K^*.

One important aspect that should be taken into account is the trade-off between the software development cost for improving the solving strategy and the monetary cost of acquiring better hardware that can provide much lower CPU times by means of faster processor and smarter memory handling techniques. In particular, nowadays linear programming solvers have been able to efficiently handle very large size problems.

However, we consider that there is still much room for the application of the proposed approach, since: 1) in the short-term scheduling problem, decisions should be made in a very short time window; therefore, it is important to solve the problem as fast as possible; 2) there are several aspects of the short-term problem that could yet be included in the problem formulation, which could cause the problem to be even larger and more difficult to be solved as a single linear program. Finally, uncertainties on the model can be considered, leading to more complex stochastic formulations as discussed in this paper.

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