Dynamical Systems 2015 — Exam
$23^{\underline{\text{rd}}}$ of January 2016 — time allowed 3h
All the questions have the same value

1) Plot the phase portraits of $(\dot{x}, \dot{y}) = (-3x - y, 5x + 4y)$ and of $(\dot{x}, \dot{y}) = (-5x + 4y, -2x + y)$ and discuss the stability of their equilibria.

2) Plot the phase portrait of the differential equation in $\mathbf{R}^2$ equivalent to $\ddot{x} = x^3 - x$ and describe $\omega(0, \sqrt{2})$, $\alpha(0, \sqrt{2})$.

3) Show that the coordinate axes $x = 0$ and $y = 0$ are invariant under the flow of $(\dot{x}, \dot{y}) = (x - x^2 + xy, xy - 2y)$, discuss the stability of its equilibria and plot its phase portrait assuming the equation has no periodic solutions that are not constant.

4) Show that if the set $Q = \{(x, y) \in \mathbf{R}^2 : \ -1 \leq x \leq 1, \ -1 \leq y \leq 1\}$ is positively invariant under the flow of a certain C^∞ vector field v in $\mathbf{R}^2$, and if $h : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ is a C^∞ diffeomorphism, then $h(Q)$ is positively invariant under the flow of the vector field $w = h_* v$.

5) Let $f : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ be the linear map represented by the matrix $A = \begin{pmatrix} 5/6 & 1 \\ 0 & 7/6 \end{pmatrix}$. Plot the trajectories of the points $(1, 1)$, $(-1, 0)$ and $(1, -1)$ and identify the invariant subspaces E^s and E^u.