Dynamical Systems
horseshoe

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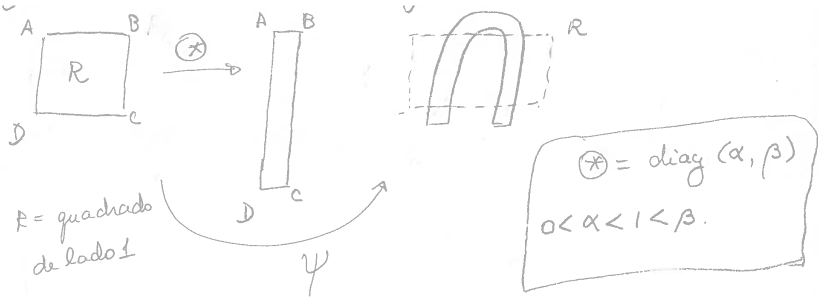
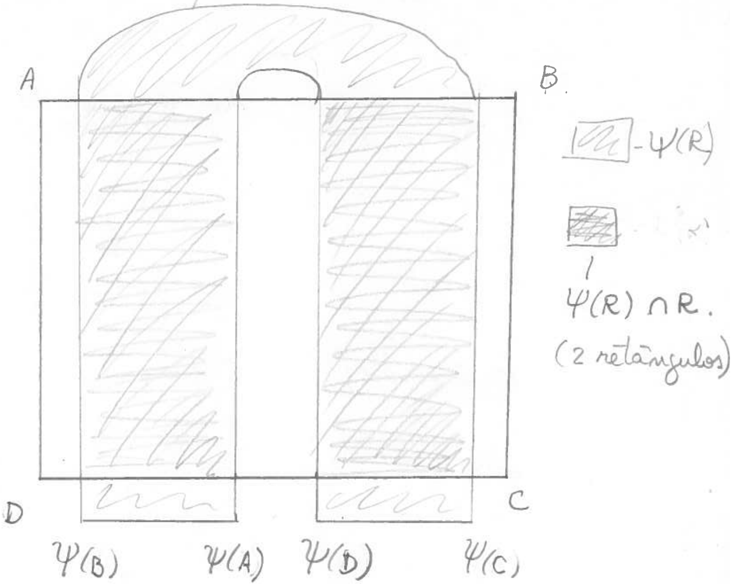
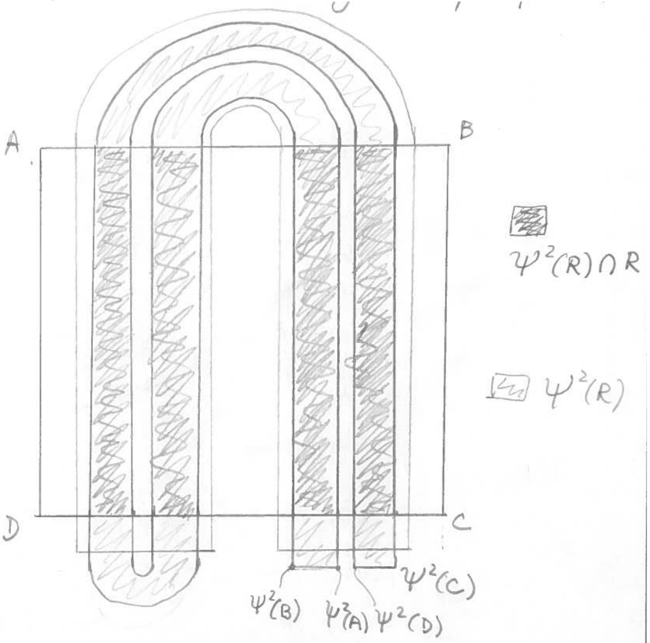
Horseshoe map $\Psi : R \longrightarrow R$ $\quad R = [0,1] \times [0,1]$



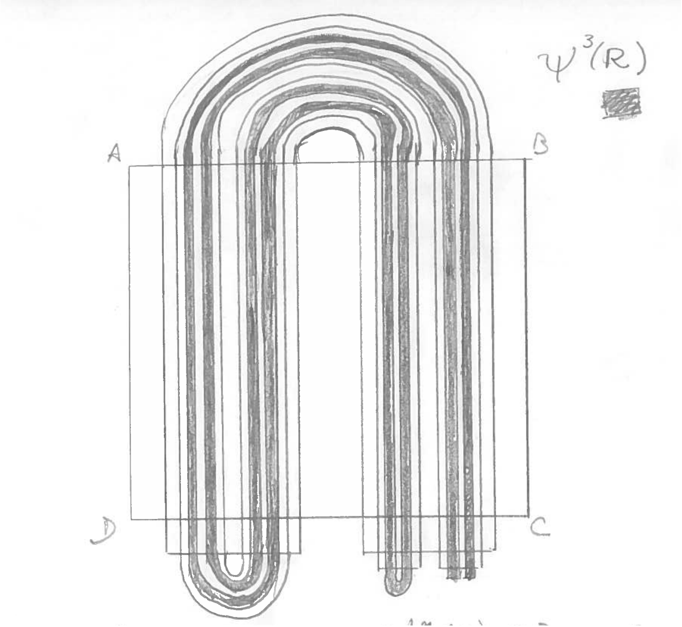
horseshoe map $\Psi : R \longrightarrow R$



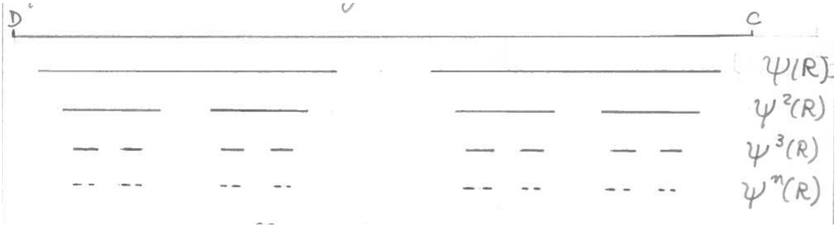
horseshoe map Ψ^2



horseshoe map Ψ^3

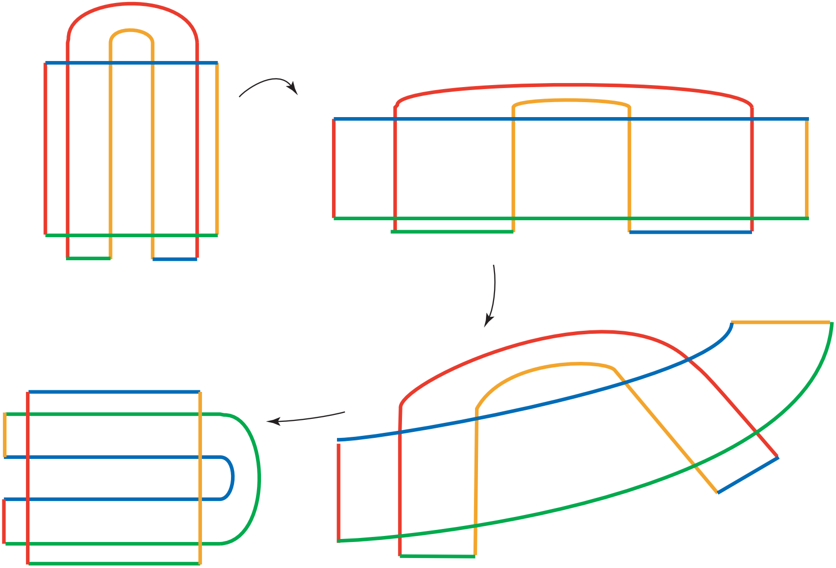


horseshoe map $\Psi^n(R) \cap \overline{CD} = \mathcal{C} \times \{0\}$



$\mathcal{C} =$ Cantor set

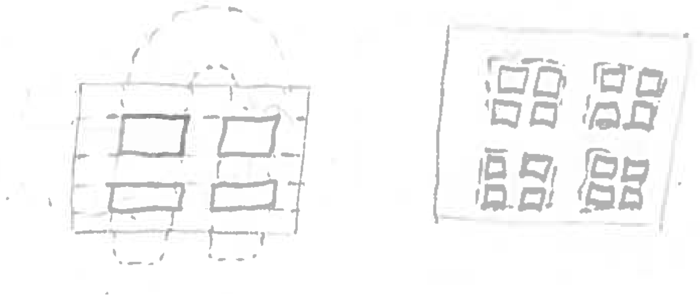
inverse



non wandering set

$$\Omega = \{(x,y) \in R: \ \Psi^n(x,y) \in R \ \forall n \in \mathbf{Z}\} = \bigcap_{n \in \mathbf{Z}} \Psi^n(R)$$

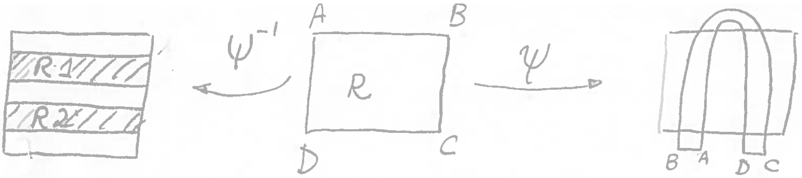
$$\Omega \subset R \cap \Psi(R) \cap \Psi^{-1}(R)$$

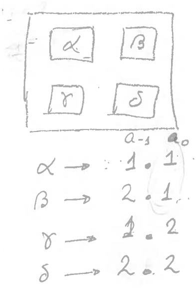


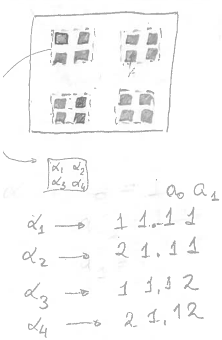
$$\Omega \subset R \cap \Psi(R) \cap \Psi^{-1}(R) \cap \Psi^2(R) \cap \Psi^{-2}(R)$$

symbolic dynamics

$$p \in \Omega \qquad a_j(p) = \begin{cases} 1 & \text{if } \Psi^j(p) \in R_1 \\ 2 & \text{if } \Psi^j(p) \in R_2 \end{cases} \qquad j \in \mathbf{Z}$$







symbolic dynamics

$$p \in \Omega \qquad a_j(p) = \begin{cases} 1 & \text{if } \Psi^j(p) \in R_1 \\ 2 & \text{if } \Psi^j(p) \in R_2 \end{cases} \qquad j \in \mathbf{Z}$$

This induces a bijection: $\Omega \longrightarrow \{1,2\}^{\mathbf{Z}}$

$$a_j(\Psi(p)) = a_{j+1}(p)$$

Consequences

- Periodic orbits of all periods.
- Orbits that are dense in Ω.
- Chaos!