$10^{\underline{\text{th}}}$ International Workshop on Real and Complex Singularities

São Carlos, July 2008

On the occasion of the $60^{\underline{\text{th}}}$ birthdays of
Maria Aparecida Ruas
and
Terence Gaffney

Cidinha

Cidinha's work

The kernel of Cidinha's work lies in the problem of classification of singularities, specially those finitely determined.

From her work in this problem we can distinguish three main directions:

- Topological and Differential Classification of Singularities.
- Study of the triviality and equisingularity of families of maps.
- Applications to the Extrinsic Geometry of Submanifolds (Generic Geometry) and Dynamical Systems.

Master thesis, 1974

Supervisor: G. Loibel.

Problem

Classification of germs of functions according to the order of determinacy.

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Main Result

Independent determination of Arnol'd series A_k and D_k.

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That is not Cidinha in the picture!

first joint work...

J.G. Ruas Filho and M.A.S. Ruas, Janaína

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Brown University period

Supervisor: T. Gaffney

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Are the 1-modal families of finitely determined germs appearing in the boundary of Mather's nice dimensions topologically stable?

(J. Damon). Then — Study of problems in finite determinacy.

Brown University period: initial work on Generic Geometry

In parallel, Cidinha attends a seminar with T. Banchoff, T. Gaffney and C. Mc Crory on Porteous' work on the generic singularities of distance squared functions. She proposes to study the analogous problem for projections of surfaces in $\mathbb{R}^3$ into planes.

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In collaboration with Gaffney she starts to study this problem. They get the classification of all the germs of codimension ≤ 2 of projections of a surface onto the plane. They show that, for a residual subset of surfaces in $\mathbb{R}^3$, the family of projections onto planes is locally versal and provide the geometrical interpretation of the singularities (unpublished work).

Doctoral thesis, 1983

Problem: Geometry and Classification of singularities.

Supervisor: L. Fávaro.

Cidinha uses infinitesimal methods of classification of singularities and controlled vector fields to address the following problems:

1. Estimates of the order of determinacy (participation of L. Wilson)

An example of result is the following:

Theorem

If $f:(k^n,0)\rightarrow(k^p,0)$ satisfies convenient Lojasiewicz conditions ($\mathcal{G}$-condition), and the coordinate functions are homogeneous, then f is $(r-l+1)-C^l-\mathcal{G}$- determined, $1\leq l\leq\infty$.

M.A.S. Ruas On The Degree Of C^L-Determinacy, *Mathematica Scandinavica* 1986

Doctoral thesis, 1983

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1. Estimates of the order of determinancy (participation of L. Wilson)

These results were extended for quasihomogeneous germs.

M.A.S. Ruas, M. Saia C^L-Determinacy Of Weighted Homogeneous Germs, *Hokkaido J. of Maths.* 1997

Doctoral thesis, 1983

2. Topological triviality results for the group $\mathcal{A}$.

3. Motivated by A. du Plessis work on the determinacy of smooth maps (Invent. Math. 1980), Cidinha proposes a method for the $\mathcal{A}$-classification of singularities, that relies into the consideration of the structure of the $\mathcal{A}$-orbits inside the $\mathcal{K}$-orbits:

She obtains sufficient conditions for the existence of open $\mathcal{A}$-orbits in a given contact class.

The existence of open $\mathcal{A}$-orbits is a necessary condition for the existence of $\mathcal{A}$-simple germs in a given $\mathcal{K}$-orbit.

Doctoral thesis, 1983

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This has been used in later works on the classification of $\mathcal{A}$-simple germs as, for instance:

Doctoral thesis, 1983

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J.H. Rieger, M.A.S. Ruas, Classification of $\mathcal{A}$-simple germs from K^n to K^2, $n \geq p$, *Compositio Mathematica* 1991

other joint work at the time

- J.G. Ruas Filho and M.A.S. Ruas, José
- J.G. Ruas Filho and M.A.S. Ruas, Juliana

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Topological and Differential Classification of Singularities

Marar and Mond studied the topology of finitely determined germs $f : (\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$, and showed that for a stable perturbation f_t of f, the number of Whitney umbrellas of f_t, the number of triple points of f_t and the Milnor number of the curve of double points of f are analytical invariants of the germ f. These invariants may be computed as the dimension of algebras associated to the singularities.

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Question (Mond)

Given a family of $\mathcal{A}$-finitely determined germs $f_t : (\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$ such that the invariants above are constant, is the family f_t topologically trivial?

Topological and Differential Classification of Singularities

First result, for a special class of germs from $\mathbb{C}^2$ to $\mathbb{C}^3$, in

M.A.S. Ruas, On equisingularity of families of corank 1 generic germs, *Contemporary Maths.* 1994

Based on Gaffney's results on equisingularity of families of $\mathcal{A}$-finite germs, Cidinha has shown that for a generic family of germs with isomorphic local algebras:

i) D. Mond's invariants are constant along the family,

ii) the family is Whitney equisingular.

Topological and Differential Classification of Singularities

With R. Bedregal and K. Houston, Cidinha answers Mond's question in the affirmative, as a consequence of a characterization of the equisingularity of families in two and three dimensions:

Theorem

Topological triviality and Whitney equisingularity are equivalent notions for families of finitely determined germs in dimensions 2 and 3. Moreover the Milnor number of the curve of double points in the source is complete invariant for equisingularity.

R. Bedregal, K. Houston and M.A.S. Ruas, Topological triviality of families of surfaces in $\mathbb{C}^3$, preprint, 2006

Topological and Differential Classification of Singularities

0-stable invariants for weighted homogeneous, $\mathcal{A}$-finite germs of corank 1, $f : \mathbb{C}^n \to \mathbb{C}^n$, were studied by W.L. Marar, J. Montaldi and Cidinha, who found formulæ for the number of isolated multiple points, in terms of the weights and quasi-homogeneity degree of the germ.

W.L. Marar, J. Montaldi and M.A.S. Ruas, Multiplicities of zero-schemes of corank 1 quasihomogeneous singularities, Singularity Theory, Cambridge University Press, 1999

Maximal Deformations

A related problem in the real case is:
Existence of stable perturbations realizing the 0-stable invariants appearing in the complex case (M-deformations).

Previous results:

- Gusein-Zade (1975) and A'Campo (1975) (independently) treated the case of germs of plane curves, proving that M-deformations always exist.
- Arnol'd (1992) and Entov (1993) showed the existence of M-deformations for singularities of type A_k and D_k.

Maximal Deformations

With J. Rieger and R.G. Wik Atique, Cidinha proved the existence of M-deformations for $\mathcal{A}$-simple germs of corank 1, in the dimensions (n, p), $n \geq p$, and $(n, n+1)$, for all $n \neq 4$.

These results are the consequence of new classification techniques that refine those of Cidinha's thesis. They provide necessary conditions for the simplicity of map germs and the analysis of the existence of real M-deformations of these singularities.

- J.H. Rieger and M.A.S. Ruas, *M-deformations of $\mathcal{A}$-simple Σ^{n-p+1}-germs from $\mathbb{R}^n$ to $\mathbb{R}^p$, $n \geq p$*, *Math. Proc. Cambridge Phil. Soc.* 2005
- J.H. Rieger, M.A.S. Ruas and R.G. Wik-Atique, *M-deformations of $\mathcal{A}$-simple germs from $\mathbb{R}^n$ to $\mathbb{R}^{n+1}$*, *Math. Proc. Cambridge Phil. Soc.* 2008
- J.H. Rieger, M.A.S. Ruas and R.G. Wik-Atique, Real deformations and invariants of map-germs, *Banach Center Publications*

Singularities of Real Analytic Varieties

Results on topological triviality and equisingularity of families of sections of analytical varieties were obtained with J. Tomazella.

- M.A.S. Ruas and J.N. Tomazella, An infinitesimal criterion for topological triviality of families of sections of analytic varieties, Proc.of MSJ- IRI Symposium "Singularity theory and its applications"
- M.A.S. Ruas and J.N. Tomazella, Topological triviality of families of functions on analytic varieties, *Nagoya Math. J.* 2004

Singularities of Real Analytic Varieties

With J. Seade, A. Verjovsky and R.N.A. dos Santos, Cidinha obtains results connected to those of King on the study of the topology of Milnor fibrations associated to germs of analytic maps with isolated singularities.

- M.A.S. Ruas, J. Seade and A. Verjovsky, On real singularities with a Milnor fibration, Trends in singularities, Birkhäuser 2002
- M.A.S. Ruas and R.N.A. dos Santos, Real Milnor Fibrations and (C)-Regularity, *Manuscripta Mathematica* 2005

Bi-Lipschitz Triviality

With A.C.G. Fernandes, L. Birbrair and J.C.F. Costa Cidinha obtained:

- A sufficient condition for bi-Lipschitz triviality of deformations of quasihomogeneous map-germs with an isolated singularity at the origin.
- invariants for bi-Lipschitz contact equivalence of germs.

A.C.G. Fernandes and M.A.S. Ruas, Bi-Lipschitz determinacy of quasihomogeneous germs, *Glasg. Math. J.* 2004

A.C.G. Fernandes, L. Birbrair, J.C.F. Costa and M.A.S. Ruas, K-bilipschitz equivalence of real function germs, *Proceedings of the American Mathematical Society* 2007

Euler Obstruction

A project with J.P. Brasselet (I.M. Luminy) and N.G. Grulha Junior (post-doc) is the study of the Euler obstruction associated to singular varieties that are discriminant sets of finitely determined germs $f : \mathbb{C}^n, 0 \to \mathbb{C}^p, 0$.

Generic Geometry

• With D.K. Hayashida Mochida and M.C. Romero-Fuster, Cidinha gets a geometrical description of the singularities of the family of height functions on surfaces immersed in $\mathbb{R}^4$. This leads to the introduction of binormal directions on these surfaces and to a new characterization of asymptotic directions and inflection points. The convexity of a surface in 4-space is also characterized in terms of existence of asymptotic directions.

D.K.H. Mochida, M.C. Romero-Fuster and M.A.S. Ruas, The geometry of surfaces in 4-space from a contact viewpoint, *Geometriae Dedicata* 1995

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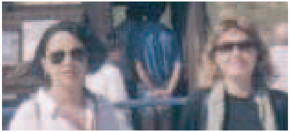
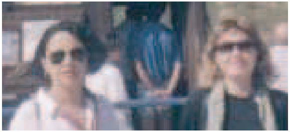
This starts the work on Generic Geometry of submanifolds of codimension higher than one in Euclidean space. It is the first of a series of works extending these methods to higher codimensions and to submanifolds of Hyperbolic and Lorentz-Minkowski spaces by several authors.



Generic Geometry

• With R. Garcia, D.K. Hayashida Mochida and M.C. Romero-Fuster, Cidinha studies the isolated singularities of the differential implicit equation of the asymptotic lines on a generic surface in $\mathbb{R}^4$. This leads to the obtaining lower bounds for the number of inflection points in terms of the topology of closed surfaces.

R. Garcia, D.K.H. Mochida, M.C. Romero-Fuster and M.A.S. Ruas, Inflection points and topology of surfaces in 4-space, *Trans. Amer. Math. Soc.* 2000



Generic Geometry

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The results obtained in this work extend Feldman's results on the number of umbilics of generic surfaces in 3-space and lead to the formulation of a generalized form for Caratheodory's and Loewner's Conjectures. These results may be seen as a generic proof of these generalized conjectures. Extensions to certain non necessarily generic cases have been later obtained by C. Gutierrez and Ruas, and J.J. Nuño-Ballesteros.

Generic Geometry

• With D.K. Hayashida Mochida and M.C. Romero-Fuster, Cidinha extends the concept of binormal directions to submanifolds immersed with codimension 2 in Euclidean spaces, providing also a characterization of the asymptotic directions in terms of height functions. Necessary conditions for convexity and sphericity of these immersions in terms of the global behaviour of the asymptotic directions are also obtained.

D.K.H. Mochida, M.C. Romero-Fuster and M.A.S. Ruas, Osculating hyperplanes and asymptotic directions of codimension two submanifolds of euclidean spaces, *Geometriae Dedicata* 1999

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This has been the basis for further works on global extrinsic properties of submanifolds immersed with codimension two, (concerning the characterization of semiumbilicity and of flatnes of the normal bundle) by M.C. Romero-Fuster with F. Sánchez-Bringas and with J.J. Nuño-Ballesteros.

Generic Geometry

• With D.K. Hayashida Mochida and M.C. Romero-Fuster, Cidinha introduces the study of asymptotic direction fields on surfaces immersed in $\mathbb{R}^5$, analyzing their generic existence and showing that the singularities of these fields are 2-singular points in the sense of Feldman.

•With M.C. Romero-Fuster and F. Tari, Cidinha determines the equation of the asymptotic curves in terms of the coefficients of the second fundamental form and studies their generic local configurations.

These techniques provide a new tool in the open problem of the existence of 2-singular points on generic surfaces in $\mathbb{R}^5$.

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D.K.H. Mochida, M.C. Romero-Fuster and M.A.S. Ruas, Inflection points and non-singular embeddings of surfaces in R^5, *Rocky Mountain J. Math.* 2003

M.C. Romero-Fuster, M.A.S. Ruas and F. Tari, Asymptotic lines of surfaces in $\mathbb{R}^5$. *Communications in Contemporary Mathematics* 2008.

Applications to Dynamical Systems

With C. Gutierrez, Cidinha obtains a determinacy condition for the index of a vector field on the plane at an isolated singularity, in terms of Newton polyhedra for the vector field. This was used to study generically embedded surfaces in R^4.

With A.C. Nabarro, Cidinha continues this study. They obtain sufficient conditions ensuring that the maximal absolute value of the degree of a planar vector field is attained.

M.A.S. Ruas and C. Gutierrez, Indices of Newton non-degenerate vector fields and a conjecture of Loewner for surfaces in R^4, Lecture Notes in Pure and Applied Mathematics, 232, 2003

A.C. Nabarro and M.A.S. Ruas, Vector fields in R^2 with maximal index, *Quart. J. Mathematics* 2008

Applications - Bifurcation

- Bifurcation problems with symmetry in parameter space - with A.M. Sitta and J.E. Furter

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- Bifurcation problems with symmetry in parameter space - with A.M. Sitta and J.E. Furter
- Singularities in nerve impulse equations - with I.S.Labouriau
  The general formulation of the problem has started a new line in Mathematical Biology. The work continues, by I.S. Labouriau with other collaborators - C.M. Rito, H.M. Rodrigues, C.M. Alves-Pinto, P.R.F. Pinto.

I.S. Labouriau and M.A.S. Ruas Singularities of equations of Hodgkin-Huxley type, *Dynamical Systems* 1996

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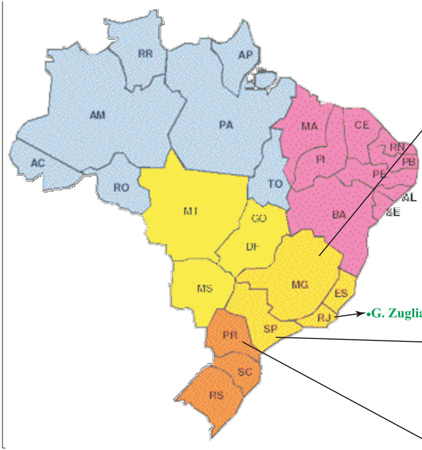

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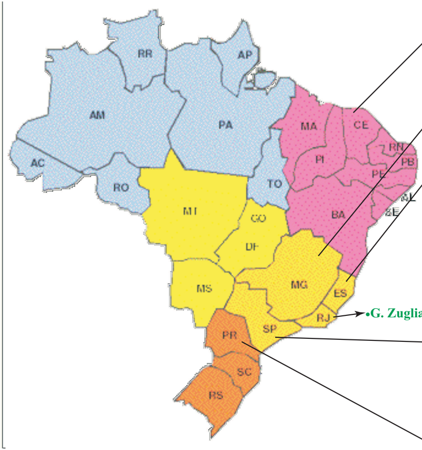

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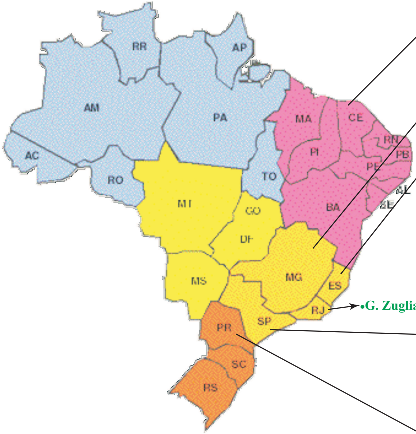
MSc, Ph.D., Post-Doc Students



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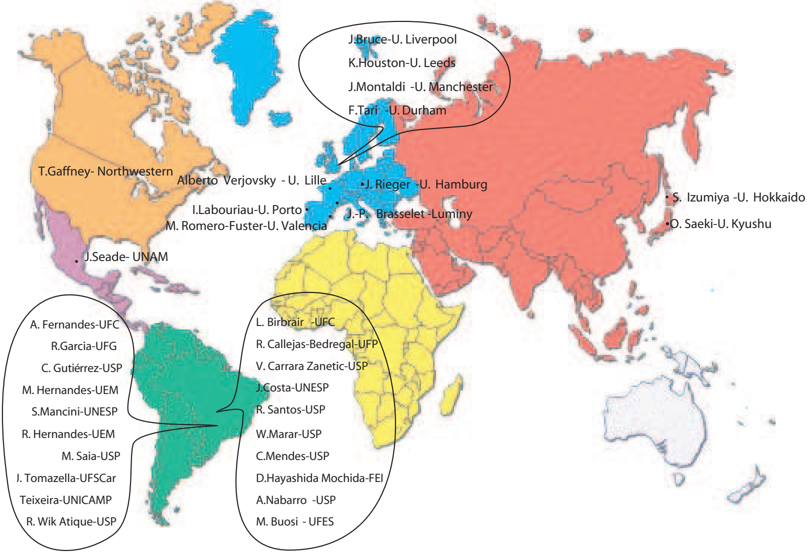

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José Gaspar Ruas and.....

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