Dynamics near heteroclinic networks with saddle-foci

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COMPETE

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Dynamics near heteroclinic networks

Joint work with

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  Faculdade de Ciências da Universidade do Porto

also based on work with

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  Faculdade de Economia da Universidade do Porto
- Sofia B.S.D. Castro
  Faculdade de Economia da Universidade do Porto

Outline

Context: smooth vector fields on $\mathbf{S}^3$

Heteroclinic cycle

Finite set of flow-invariant equilibria (nodes)

trajectories joining them (connections) in a cycle

Heteroclinic network

Connected set, union of heteroclinic cycles.in a cycle

Heteroclinic cycles and networks are not persistent.

Special context for persistent heteroclinic cycles:
equations with symmetry (flow-invariant subspaces)

Network questions — dynamics around the network

- switching;
- trajectories that remain near the network;
- geometric structure.

A specific type of network

Symmetries in $\mathbf{S}^3$ — inherited from $\mathbf{R}^4$

$$\gamma_1(x_1, x_2, x_3, x_4) = (-x_1, -x_2, x_3, x_4)$$

$$\gamma_2(x_1, x_2, x_3, x_4) = (x_1, x_2, -x_3, x_4)$$

In $\mathbf{S}^3$, invariant sets:

circle $\mathbf{S}^1 = \{(0, 0, x_3, x_4)\} \cap \mathbf{S}^3$

2-sphere $\mathbf{S}^2 = \{(x_1, x_2, 0, x_4)\} \cap \mathbf{S}^3$

two equilibria $\mathbf{S}^1 \cap \mathbf{S}^2 = \{(0, 0, 0, \pm 1)\}$

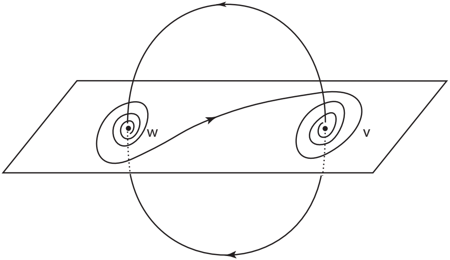
Compatible with heteroclinic network

that persists under symmetry-preserving perturbations.

Explicit examples constructed in: Aguiar, Castro, Labouriau (Int. J. Bif. Chaos 2006),

Rodrigues, Labouriau (Physica D 2014), Labouriau, Rodrigues (preprint 2014)

A specific type of network on $\mathbf{S}^3$



Two equilibria v and w

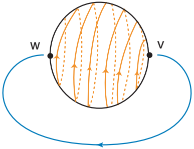
2-D connection
$$W^u(w) = W^s(v)$$

1-D connection
$$W^s(w) = W^u(v)$$

Network is asymptotically stable

From symmetry:

$$\overline{W^u(w)} = \overline{W^s(v)} = \mathbf{S}^2$$
$$\overline{W^s(w)} = \overline{W^u(v)} = \mathbf{S}^1$$



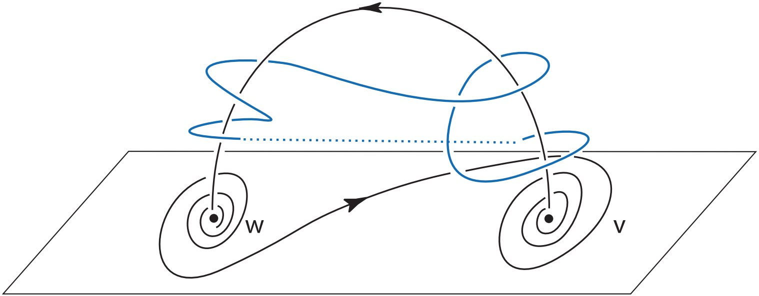
Breaking the symmetry, get:

$W^s(v) \pitchfork W^u(w)$

$W^u(v) \cap W^s(w) = \emptyset$

Two different kinds of networks of this type

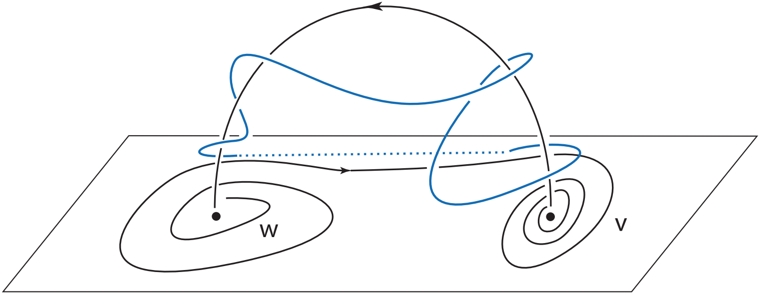
Turn with the same orientation around equilibria



Joining the end points of nearby trajectories
always links them to the network.

The nodes have the same chirality.

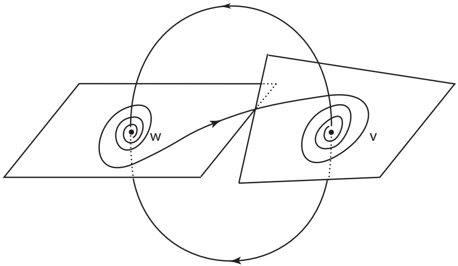
Two different kinds of networks of this type



Joining the end points of nearby trajectories may not yield a link.

The nodes have different chirality.

Transverse 2-D manifolds — any chirality



Suspended horseshoes around the network.

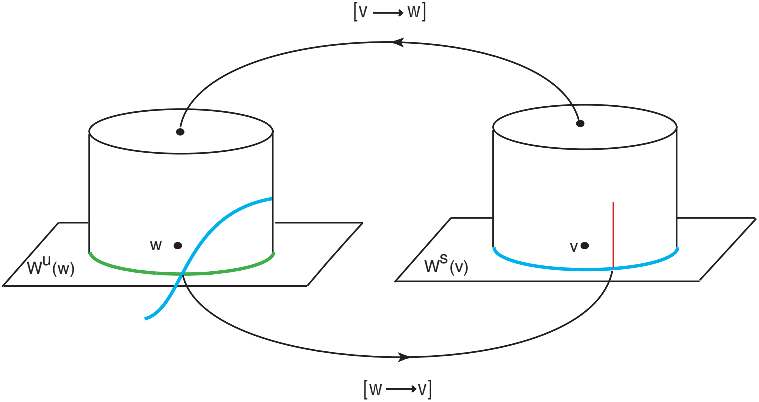
Switching near the network

Same chirality:

Hyperbolic Poincaré first return map.

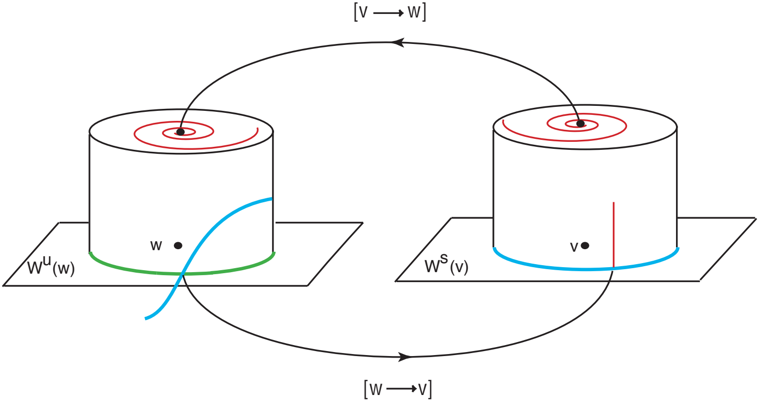
Transverse 2-D manifolds — same chirality

A segment of initial conditions is mapped into a helix.



Transverse 2-D manifolds — same chirality

A segment of initial conditions is mapped into a helix.



Transverse 2-D manifolds — same chirality

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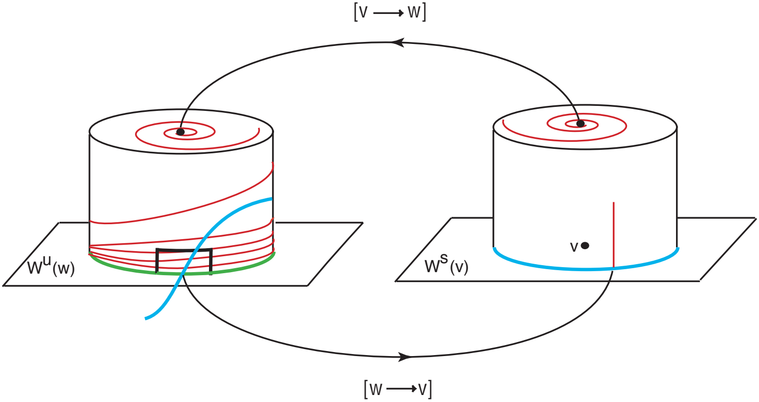

Transverse 2-D manifolds — same chirality

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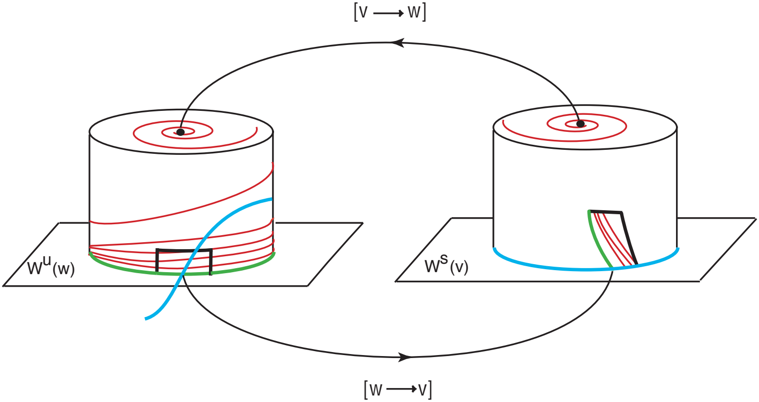
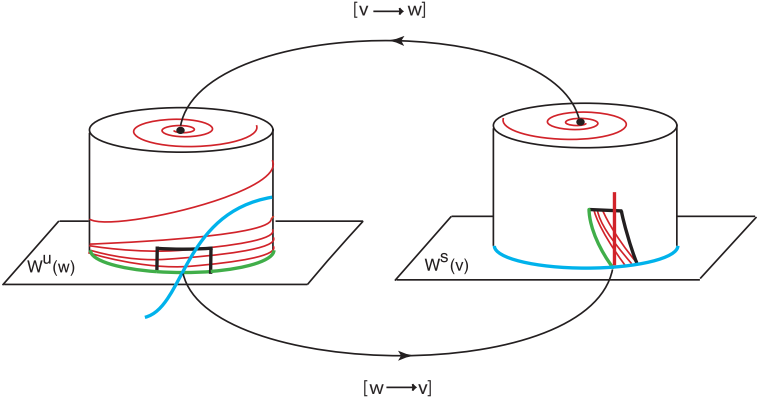


Switching

Transverse 2-D manifolds — same chirality

A segment of initial conditions is mapped into a helix.



Suspended horseshoes.

Different chirality

With 2-D connection

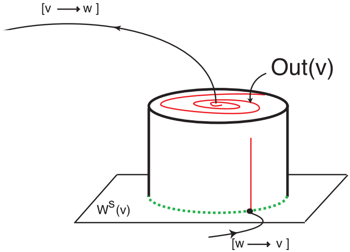
Like same chirality.

Transverse intersection of 2-D manifolds

A segment is not mapped into a helix.

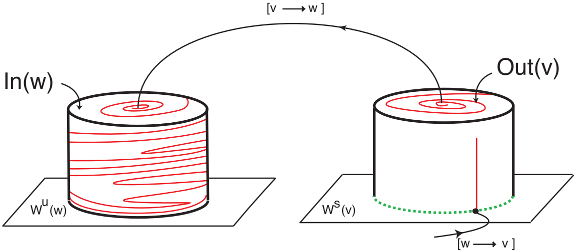
Different chirality — transverse 2-D manifolds

A segment is not mapped into a helix



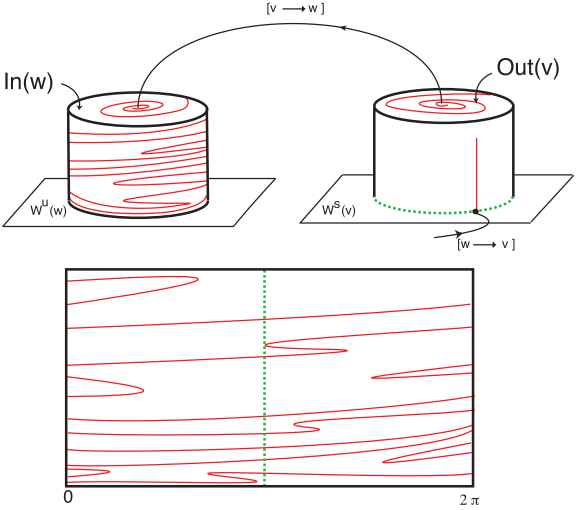
Different chirality — transverse 2-D manifolds

A segment is not mapped into a helix



Different chirality — transverse 2-D manifolds

A segment is not mapped into a helix



but its image still accumulates on $W^u(w)$.

Different chirality— transverse 2-D manifolds

For an open set of parameters

In dense subset of parameters
Heteroclinic tangencies of 2-D invariant manifolds $W^s(v)$ $W^u(w)$.
Infinitely many sinks nearby (Newhouse).

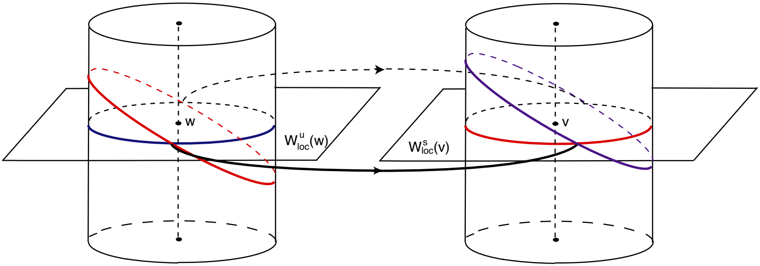
Coexisting with

Suspended hyperbolic horseshoes around the network.

All these structures form a spiralling attractor that follows the network.

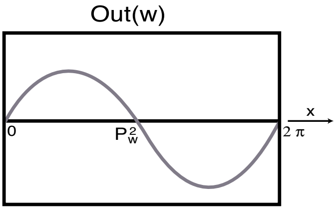
...but they correspond to a set of initial conditions with zero Lebesgue measure.

Same chirality — close to symmetry



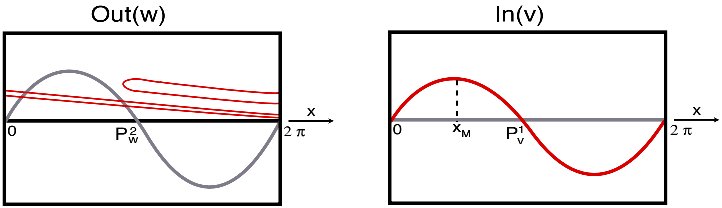
Transverse 2-D manifolds on a closed curve on the boundary of neighbourhoods of nodes.
At least two intersections of invariant manifolds.
Assume: closed curve has a single maximum (generic).

Same chirality — close to symmetry



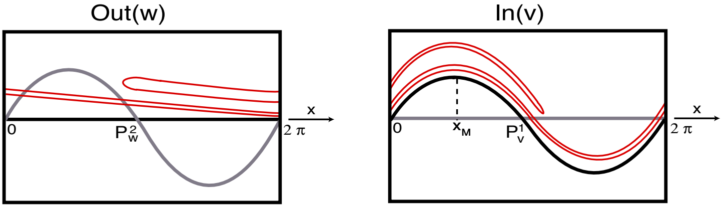
$W^u(w) \cap In(v)$ is a smooth curve with a single maximum.

Same chirality — close to symmetry



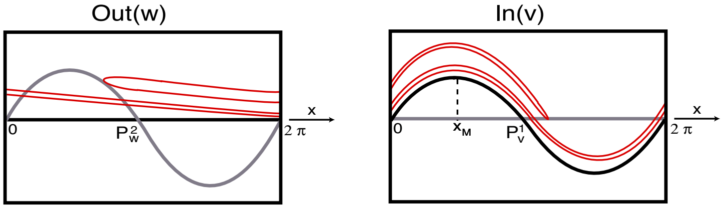
$W^u(w) \cap In(v)$ mapped into two helixes in $Out(w)$.

Same chirality — close to symmetry



$W^u(w) \cap In(v)$ mapped into two helices in $Out(w)$.

Same chirality — close to symmetry



Small perturbation approaching symmetry

$W^u(w)$ tangent to $W^s(v)$.

The End

Thank you for your attention

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