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For a detailed discussion of white noise theory via the canonical Gaussian measure and accessible random variables, and for applications of that theory to non-linear filtering, see [6]. It contains many references to the earlier literature, including references to the seminal work of I. Segal and L. Gross. Some more recent papers making use of the theory are [1], [4], [7].

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G.W. Johnson

MSC 1991: 60B11, 60H05, 60H07

ACCURACY, *accuracy analysis* – A systematic study of the precision and errors of numerical and statistical calculation and estimation procedures. See, e.g., **Error**; **Errors, theory of**.

M. Hazewinkel

MSC 1991: 62-XX, 65-XX

ACTION POTENTIAL – An electrical disturbance propagated as a wave along an axon (elongated part of a nerve cell) that is considered as the way information is transmitted in the nervous system of animals. Cardiac, muscle and some endocrine cells also display action potentials with similar properties.

An action potential is observed experimentally as a displacement of voltage from its equilibrium value that takes place in a limited part of the axon and that retains its shape while it is propagated with constant speed. It appears in response to a sufficiently large stimulus, subthreshold stimulation producing a transient departure from equilibrium that is not propagated. After the passing of an action potential the axon apparently returns to its equilibrium state but the threshold value is raised for some time, the *refractory period*.

A special experimental setting, called a *current clamp*, eliminates spatial variations and the voltage curve in this case is called a *stationary action potential*.

Action potentials are described mathematically as undamped *travelling-wave solutions* of the **Hodgkin–Huxley system**.

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I.S. Labouriau

MSC 1991: 92C20

ACTIVATION ENERGY – A concept originating in the theory of chemical reactions. It plays an important role in **combustion theory**.

The evolution of a chemical reaction is determined by its *specific-reaction rate constant*, usually denoted by k. This quantity depends mainly on the temperature, T. For a one-step chemical reaction, its functional dependence is given empirically by the *Arrhenius expression*: $k = A\exp(-E/(RT))$. Here, R is the universal gas constant, E the activation energy and A the frequency factor for the reaction step; E is independent of T, while A may depend weakly (for example, polynomially) on T.

At the molecular level, a chemical reaction is a collision process between reactant molecules, from which reaction products emerge. The molecules move on a potential-energy surface, whose shape is determined by a solution of the **Schrödinger equation**. A configuration of the reactant molecules corresponds to a local minimum in one region, and a configuration of the reaction products to a local minimum in another region, where the two minima are generally separated by a barrier in the potential-energy surface. At a **saddle point** on the barrier, the height of the surface above the energy of the reactant region assumes a minimum value. A collision of the reactant molecules can produce products only if the energy of the reactants (for example, their kinetic energy) exceeds this minimum height. The minimum barrier height defines the activation energy, E. It is the energy the reactants must acquire before they can react. In practice, the activation energy is determined experimentally, by measuring k at various values of T and making a best straight-line fit through the data $\ln k$ versus $1/T$.

Activation-energy asymptotics. Activation-energy asymptotics play an important role in **combustion theory**, in the study of diffusion flames.