Elementos de Matemática II

Soluções dos exercícios (2002–2003)

Funções reais de várias variáveis

1. (a) (b)

2. (a) 3; 6 (b) $e^{-xy}(1-xy)$; $-x^2e^{-xy}+3$ (c) $\frac{-x^2+y^2-2xy+1}{(x^2+y^2+1)^2}$; $\frac{x^2-y^2-2xy+1}{(x^2+y^2+1)^2}$
   (d) $1/3$; 0 (e) $2xz^3+y$; x; $3x^2z^2-\sec^2 z$ (f) $\sqrt{3}$; $-\sqrt{3}$; $-\frac{\sqrt{3}}{3}$ (g) 1; $\ln 4$; $\ln 4$
   (h) $2y^3$; $6x^2y-126xy^5$; $6xy^2-21y^6$ (i) -2; $-\pi$ (j) 2; 2; 0; 0

3. (a) Mínimo: $(0,0)$. Pontos de sela: $(2\sqrt{2},4)$ e $(-2\sqrt{2},4)$
   (b) $(0,\pi/2+k\pi)$, $k\in\mathbb{Z}$, é ponto de sela se k é par e mínimo se k é ímpar.
   (c) Mínimo: $(0,0)$. Ponto de sela: $(2,0)$
   (d) Mínimo: $(2,1)$. Máximo: $(-2,-1)$. Pontos de sela: $(1,2)$ e $(-1,-2)$

4. $x=y=z=2\mathrm{m}$

5. $x=y=z=1000$

6. (a) 44 (b) $\frac{220}{3}$ (c) $\frac{1}{8}(\operatorname{arctg}\frac{1}{4}-\operatorname{arctg}\frac{3}{4})+\frac{1}{3}$ (d) $\frac{1}{2}(-e+e^{-3}+e^4-1)$ (e) $-\pi-2$

7. (a) $\frac{1}{12}$ (b) $\frac{3\pi\sqrt{2}-11}{4}$ (c) 2 (d) $\frac{1}{2}(1-\cos 1)$ (e) $e^3-\frac{e^2}{2}-e+\frac{1}{e}-\frac{1}{2}$ (f) $\frac{99}{40}$ (g) $-\frac{24}{5}$ (h) $\frac{81}{8}$

8. (a) $\frac{1}{6}(e^9-1)$ (b) $\frac{2}{9}(2\sqrt{2}-1)$ (c) $e-1$ (d) $\frac{1}{12}(1-\cos 1)$

9. (a) $\frac{22}{3}$ (b) $\frac{125}{6}$ (c) $\sqrt{2}-1$

10. (a) $\frac{11}{6}$ (b) $\frac{2}{15}(19^{\frac{5}{2}}-3^{\frac{5}{2}}-4^5)$ (c) $\frac{34}{3}$

11. (a) $\frac{16}{3}$ (b) $\frac{7}{5}$ (c) $\frac{5}{28}$

Sistema de Equações Lineares e Matrizes

12. (a) $(3,-1)$ (b) $(-13/7, 44/7, -3)$ (c) $\emptyset$ (d) $(1-z, 2-z, z);\ z \in \mathbb{R}$ (e) $(2, -z, z);\ z \in \mathbb{R}$ (f) $(z-15, 6-3z, z, 2z+2);\ z \in \mathbb{R}$ (g) $\emptyset$ (h) $(1,-1,2)$ (i) $(0,0,0,0)$ (j) $(0,0,0)$

13. $\begin{pmatrix} 0 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & -4 & 3 \end{pmatrix}$; $\begin{pmatrix} 4 & 2 & 6 \\ -2 & 0 & 8 \\ 0 & -2 & 4 \end{pmatrix}$; $\begin{pmatrix} -9 & 11/2 & 3/2 \\ 9/2 & 10 & -1/8 \\ 0 & -31/2 & 6 \end{pmatrix}$

14. (a) $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ (b) $\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ (c) $\begin{pmatrix} 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ -5 \\ -4 \end{pmatrix}$ (e) $\begin{pmatrix} 1 & 8 \\ -5 & 17 \\ -4 & 10 \end{pmatrix}$ (f) $\begin{pmatrix} -3 & -5 & -1 \\ 2 & -13 & 4 \\ -1 & -8 & 6 \end{pmatrix}$ (g) $\begin{pmatrix} 2 & 11 \\ 1 & 25 \end{pmatrix}$

15. $A^{-1} = \frac{1}{5}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$; $B^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix}$

16. (a) 1 (b) 21 (c) 0 (d) 13 (e) 16 (f) 39

17. (a) -189 (b) $-1/7$ (c) $-8/7$ (d) $-1/56$ (e) 7

18. (a) $\begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$ (b) $\frac{1}{8}\begin{pmatrix} 2 & -6 & -4 \\ -1 & 3 & -2 \\ 1 & 5 & 2 \end{pmatrix}$ (c) $\frac{1}{2}\begin{pmatrix} -1 & 3 & -1 \\ 2 & -4 & 2 \\ 1 & 1 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} -3 & -1 \\ 2 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ (f) Não existe.

19. $\lambda \in \mathbb{R} \setminus \{-1, 1\}$

20. (a) 10 (b) 6 (c) 76 (d) 0

21. $a = 0$ ou $a = 2$ ou $a = -2$

22. (a) 1 (b) 1 (c) 1

23. (a) F (b) F (c) V (d) F

24. (a) F (b) V (c) F (d) V

25. (a) $(2/13, 5/13)$ (b) $(3,-1)$ (c) $(1,-3,-2)$ (d) $(-25/7, 12/7, 19/7)$

26. Sistema possível determinado se $a \neq 3$ e $b \in \mathbb{R}$;
Sistema possível indeterminado se $a = 3$ e $b = 24$;
Sistema impossível se $a = 3$ e $b \neq 24$.

27. Se $k = 2$ tem-se $S = \{(x, x) \ : \ x \in \mathbb{R}\}$. Se $k \neq 2$ tem-se $S = \{(0, 0)\}$.

28. $a = 1$ e $b = 1$; $S = \{(1 - 4y, y, 3y) \ : \ y \in \mathbb{R}\}$.

29. $m = 0$ e $n = -4$

30. (a)
(b) Se $a \in \mathbb{R} \setminus \{-1\}$ e $b \neq -a$, então $S = \{(0, 0, 0)\}$;
se $a = -1$ e $b \neq 1$, então $S = \{(0, z, z) : z \in \mathbb{R}\}$;
se $b = -a$ e $a \in \mathbb{R} \setminus \{0\}$, então $S = \{(\frac{1+a}{2}y, \ y, \ \frac{1-a}{2}y) : y \in \mathbb{R}\}$.

(c) Se $a \in \mathbb{R} \setminus \{-1\}$ e $b \neq -a$, então $S = \{(1, 2, 1)\}$;
se $a = -1$ e $b \neq 1$, então $S = \{(1, z + 2, z + 1) : z \in \mathbb{R}\}$;
se $b = -a$ e $a \in \mathbb{R} \setminus \{0\}$, então $S = \{(1 + \frac{1+a}{2}y, \ 2 + y, \ 1 + \frac{1-a}{2}y) : y \in \mathbb{R}\}$.

31. (a) (b) (c) (d)

Espaços Vectoriais

32. São subespaços vectoriais de $\mathbb{R}^3$ os conjuntos C, D, E, G e H.

33. (a) São subespaços vectoriais de $\mathbb{R}^2$ os conjuntos B, C, E, e F. (b) $\mathbb{R}^2 = B \oplus E$.

34. (a) São subespaços vectoriais de $\mathbb{R}[X]$ os conjuntos A, B, C, E, e F. (b) $B \cap C = \{a + aX^2 \mid a \in \mathbb{R}\} \leq \mathbb{R}[X]$ (c) $B \cup C$ não é subespaço de $\mathbb{R}[X]$

35. A e D.

36. (a) A, B e C (b) A, B e C

37. $\vec{u}_1 = \vec{u}_2 + 2\vec{u}_3 + 3\vec{u}_4$

38. Por exemplo:
    $A : ((1,2,2),(1,-2,0),(-1,0,-1))$
    $B : ((2,1,0),((-1,0,1))$
    $C : ((-3,1,0))$
    $D : (1+X^2,-2+X)$
    $E : ((3,7,0,5),(0,5,3,-2))$

39. (a) Por exemplo $((1,0,1,0),(0,1,1,0),(0,0,1,1))$;
    (b) Por exemplo $((1,0,1,0),(0,1,1,0),(0,0,1,1),(0,0,0,1))$;
    (c) $B_1 : (1,1,-3)$; $B_2 : (1,1,-3,0)$

40. Por exemplo $(1-2X^2, X-X^2)$.

41. (a) Não. (b) Não.

42. (a) Por exemplo $(1,0,1,0),(0,1,1,1),(0,0,1,0),(0,0,0,1))$;
    (b) Por exemplo $((1,1,1,0),(0,0,1,1),(0,2,3,0),(0,0,0,1))$;
    (c) Por exemplo $((1,0,1,0),(0,1,1,0),(0,0,1,1))$;
    (d) $B_1 : (1,1,-1,-2)$; $B_2 : (1,0,0,-1)$; $B_3 : (1,1,-1)$

43. (a) 2 (b) 2 (c) 1

44. (a) (b) Não. (c) $p_1 = \frac{5}{4}p_2 + \frac{1}{4}p_3 + 0p_4$ (única) (d) $\mathbb{R}_2[X]$; (p_2, p_3, p_4). (e) i. V; ii. V

45. (a) (b) (c)

46. (a) V (b) F (c) V (d) V (e) F (f) V

Aplicações Lineares

47. (a) Não. (b) Sim. $\mathrm{N}(f) = \{(0,0,0,0)\}$; $\mathrm{Im}(f) = \mathbb{R}^4$ (c) Não. (d) Sim. $\mathrm{N}(f) = \{(0,0)\}$; $\mathrm{Im}(f) = \mathbb{R}^2$ (e) Sim. $\mathrm{N}(f) = \{(0,y,0) : y \in \mathbb{R}\}$; $\mathrm{Im}(f) = \mathbb{R}^2$ (f) Não. (g) Sim. $\mathrm{N}(f) = \{(y-2z,y,z,2y-2z) : y,z \in \mathbb{R}\}$; $\mathrm{Im}(f) = \mathbb{R}^2$ (h) Sim. $\mathrm{N}(f) = \{0\}$; $\mathrm{Im}(f) = \mathbb{R}_2[X]$ (i) Não (j) Sim. $\mathrm{N}(f) = \{(-2y-3z,y,z) : y,z \in \mathbb{R}\}$; $\mathrm{Im}(f) = \mathbb{R}$ (k) Sim. $\mathrm{N}(f) = \{a+ax+ax^2 : a \in \mathbb{R}\}$; $\mathrm{Im}(f) = \mathbb{R}^2$

48. (a) $(-2,6,4)$, $(38,-2,8)$ (b) $(8x+17y-6z,-3y+2z,2x+2y)$ (c) $\mathrm{N}(f) = \{(x,-x,-\frac{3}{2}x) : x \in \mathbb{R}\}$; $\mathrm{Im}(f) = \{(x,y,z) \in \mathbb{R}^3 : x+3y-4z = 0\}$

49.

50. 3

51. Não.

52. (a) (b) $(0,0,0)$ (c) $\begin{pmatrix} 0 & 0 \\ 1 & 4 \end{pmatrix}$ (d) 1

53. (a) $(-1,6,1)$ (b) $(\,(0,-1,1)\,)$, por exemplo.

54. (a) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 & -1 & -2 \\ -2 & 0 & -3 \\ 3 & 2 & 5 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & -1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$ (e) $\begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 2 \\ 0 & 1 & 0 \end{pmatrix}$

55. (a) F (b) F (c) F (d) V (e) V (f) F (g) F (h) V

56. Polinómio característico: $\lambda^2 - 4\lambda + 3$; valores próprios: 1 e 3;
$\vec{v_1} = (x,x) : x \in \mathbb{R} \setminus \{0\}$; $\vec{v_3} = (x,-x) : x \in \mathbb{R} \setminus \{0\}$.

57. Por exemplo, $(\,(1,1,1),(1,-2,1),(1,0,-1);)$.

58. Valores próprios: $1,-1,2$;
$\vec{v_1} = (3z,2z,z) : z \in \mathbb{R} \setminus \{0\}$; $\vec{v_{-1}} = (x,0,x) : x \in \mathbb{R} \setminus \{0\}$; $\vec{v_2} = (z,3z,z) : z \in \mathbb{R} \setminus \{0\}$.

59. (a) Valores próprios: $1,2,\sqrt{2},-\sqrt{2}$;
$\vec{v_1} = (0,0,0,t) : t \in \mathbb{R} \setminus \{0\}$; $E_1 = \{(0,0,0,t) : t \in \mathbb{R}\}$;
$\vec{v_2} = (0,0,t,t) : t \in \mathbb{R} \setminus \{0\}$; $E_2 = \{(0,0,t,t) : t \in \mathbb{R}\}$;
$\vec{v_{\sqrt{2}}} = ((1+\sqrt{2})y,y,0,0) : y \in \mathbb{R} \setminus \{0\}$; $E_{\sqrt{2}} = \{((1+\sqrt{2})y,y,0,0) : t \in \mathbb{R}\}$;
$\vec{v_{-\sqrt{2}}} = ((1-\sqrt{2})y,y,0,0) : y \in \mathbb{R} \setminus \{0\}$; $E_{\sqrt{2}} = \{((1-\sqrt{2})y,y,0,0) : t \in \mathbb{R}\}$.
(b) Valor próprio: 1; $\vec{v_1} = a : a \in \mathbb{R} \setminus \{0\}$; $E_1 = \mathbb{R}$