XXIst Oporto Meeting

Loop Topological Complexity

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Let's Go : Love and Peace

بِسمِ اللَّهِ الرَّحمَنِ الرَّحِيمِ

وَ عَلَى اللَّهِ فَليَتَوَكَّلِ المُتَوَكِّلُون

صَدَقَ اللَّهُ العَظِيم

Content

1. Motion Planning Algorithms (MPA)
2. Loop Motion Planning Algorithms (LMPA)

Motion Planning Algorithms

Farber (2003)

Let X a path configuration space, a motion planning algorithm consists to produce continuously from a pair of point $(A, B) \in X \times X$ (as an input), a path in X (as an output) from A to B. Roughly speaking, that is any continuous section $s : X \times X \to PX$ of the projection

$$\begin{array}{rcccl} \pi & : & PX & \longrightarrow & X \times X \\ & & \gamma & \longmapsto & (\gamma(0), \gamma(1)) \end{array} .$$

Motion Planning Algorithms

Farber (2003)

MPA exists iff X contractible

Motion Planning Algorithms

On the sequel

X contractible, and $\mathcal{M}(X)$ denotes the non empty set of all motion planning algorithms on X topologized (as a subset of $\mathrm{map}(X \times X, PX)$) with the compact-open topology.

Derfoufi, M. (2015)

$\mathcal{M}(X)$ is also contractible

Motion Planning Algorithms

Open Question 1

- If X and Y are homeomorphic, $\mathcal{M}(X)$ and $\mathcal{M}(Y)$ are also homeomorphic ?

Motion Planning Algorithms

Partial Answer (compact manifold context)

- From Urysohn metrization theorem X is metrizable, whenever X is a compact manifold ;
- Let d be a such metric on X, it induces a metric $\mathcal{M}(d)$ on $\mathcal{M}(X)$ defined for any $s, s' \in \mathcal{M}(X)$ by

$$\mathcal{M}(d)(s, s') := \sup_{\substack{(A,B)\in X^2 \\ 0\leq t\leq 1}} d(s(A,B)(t), s'(A,B)(t));$$

- The open-compact topology of $\mathcal{M}(X)$ is the same than that induced by the metric $\mathcal{M}(d)$.

Motion Planning Algorithms

Derfoufi, M. (Classification Theorem, 2015)

(X, d_X) and (Y, d_Y) are isometric, then $(\mathcal{M}(X), \mathcal{M}(d_X))$ and $(\mathcal{M}(Y), \mathcal{M}(d_Y))$ are also.

Sketch of the proof

- To any isometry $\varphi : (X, d_X) \longrightarrow (Y, d_Y)$, we associate the isometry

$$\begin{array}{rcccc} \mathcal{M}(\varphi) & : & (\mathcal{M}(Y), \mathcal{M}(d_Y)) & \longrightarrow & (\mathcal{M}(X), \mathcal{M}(d_X)) \\ & & s & \longmapsto & \varphi^{-1} \circ s \circ (\varphi \times \varphi) \end{array}$$

Motion Planning Algorithms

A contravariant functor

$$\mathcal{M} : \left\{ \begin{array}{ll} X : & \text{contractible} \\ & \text{path-conn.} \\ & \text{comp. mfld} \\ d : & \text{metric} \\ \varphi : & \text{Isometrie} \end{array} \right\} \longmapsto \left\{ \begin{array}{lc} \mathcal{M}(X) : & \text{metrizable} \\ & \text{space} \\ & \\ \mathcal{M}(d) : & \text{metric} \\ \mathcal{M}(\varphi) : & \text{Isometrie} \end{array} \right\}$$

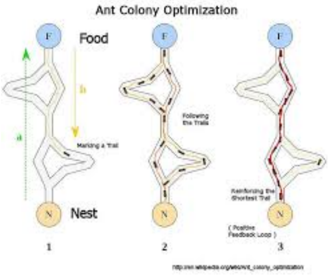
Motion Planning Algorithms

Derfoufi, M. (2015)

$(\mathcal{M}(X), \delta)$ is complete.

Interpretation

For a fixed pair of points (departure, target), then from any wider class of closed motion planning algorithms, emerges one (their limit).



Example from Nature

Ant colony optimization algorithm based on the behaviour of ants seeking for an optimal path nest-food

Motion Planning Algorithms

Open Question 2

$(\mathcal{M}(X), \delta)$ is compact ?

Interpretation

The compactness of $(\mathcal{M}(X), \delta)$ means that if one fix a pair of points (departure, target), then from any wider class of motion planning algorithms (not necessary closed), emerges one (their limit).

Loop Motion Planning Algorithms

Derfoufi, M. (2015)

Loop Motion Planning Algorithm (LMPA) : any continuous section $s : X \times X \to X^{S^1}$ of the loop evaluation

$$\begin{array}{cccc} ev : & X^{S^1} & \longrightarrow & X \times X \\ & \gamma & \longmapsto & (\gamma(0), \gamma(\frac{1}{2})) \end{array} .$$

Interpretation

- Input : a pair of points (departure, target) ;
- Output : a goings and comings motion by requiring the robot a come-back to it departure point.

Loop Motion Planning Algorithms

Application areas

- The motion of an drone like an unmanned warplane or a guided TV camera ;
- The famous vehicle routing problem with pick-up and delivery

Derfoufi, M. (2015)

LMPA exits iff X contractible.

Loop Motion Planning Algorithms

On the sequel

X contractible, and $\mathcal{M}^{\mathrm{LP}}(X)$ denotes the non empty set of all loop motion planning algorithms on X topologized (as a subset of $\mathrm{map}(X \times X, X^{S^1})$) with the compact-open topology.

Derfoufi, M. (2015)

$\mathcal{M}^{\mathrm{LP}}(X)$ is also contractible.

Loop Topological Complexity

Derfoufi, M. (2015)

$\mathrm{TC^{LP}(X)}$: the minimal number k (or infinity) such that $X \times X$ can be recovered by k open sets $U_1, \dots U_k$ such that on each of which there exists a continuous loop motion planning algorithm, s_i^{LP} over U_i verifying $s_i^{LP}(A_i, B_i)(1) = A_i$ for any $(A_i, B_i) \in U_i$.

Derfoufi, M. (2015)

$$\mathrm{TC^{LP}(X) = TC(X)}$$

Loop Motion Planning Product

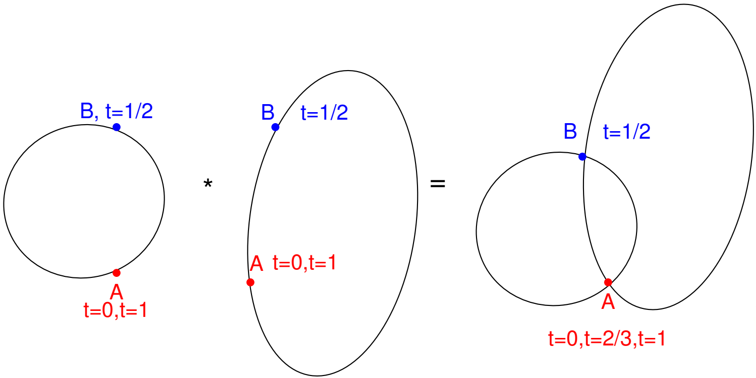
Derfoufi, M. (2015)

For any $(s_1, s_2) \in \mathcal{M}^{\mathrm{LP}}(X) \times \mathcal{M}^{\mathrm{LP}}(X)$, we put :

$$\begin{aligned} s_1 \star s_2(a,b)(t) &= s_1(a,b)(t) && \text{if } 0 \leq t \leq \tfrac{1}{2} \\ &= s_1(a,b)(3t-1) && \text{if } \tfrac{1}{2} \leq t \leq \tfrac{2}{3} \\ &= s_2(a,b)(3t-2) && \text{if } \tfrac{2}{3} \leq t \leq 1 \end{aligned}$$

Loop Motion Planning Product

Two loop motion planning algorithms are composable if and only if they have two common base points.



Open Question 3

How one may use the loop motion product or interpret the equality $\mathrm{TC} = \mathrm{TC}^{\mathrm{LP}}$?

Research Project

Key Idea

loop motion product leads to a string motion product in the same spirit of that of Chas-Sullivan on $\mathcal{M}^{\mathrm{LP}}(X)$

Main Obstacle

$\mathcal{M}^{\mathrm{LP}}(X)$ is contractible, its homology is trivial.

Research Project

Global Question

Is there any way to define global LMPA for a non contractible compact manifold

References

- Y. Derfoufi, M., Motion planning algorithms, topological proprieties and affine approximation, submitted on December 2014
- Y. Derfoufi, M., On the motion of a mobile robot in a contractible, path connected and compact manifold, final touches

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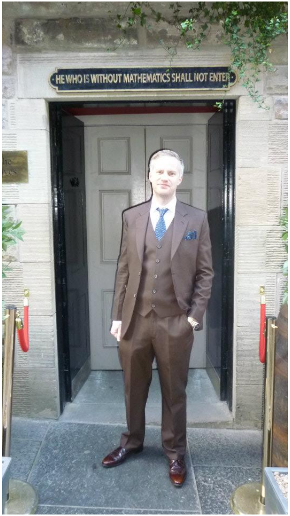

Acknowledgements

MAAT, Moroccan Research Group

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MAAT, Moroccan Research Group

- MAAT : Moroccan Area of Algebraic Topology (Born : 2012) ;
- Logo : and Home Page : http ://algtop.net
- Members : 3 professors, 7 PhD Students, 30 Master Students ;
- Scientific production
  1. 1 published papers, 7 submitted since November 2013 ;
  2. Monthly seminar
  3. Bi-Annual Research School :Geometry, Topology in Physics and Mathematics
  4. GeToPhyMa-2016 (July, http ://algtop.net/geto16) : On Rational Homotopty Theory and its Interactions, will be dedicated to D. Sullivan and J. Stasheff.

Acknowledgements

Kind Portuguese People

Questions or Comments are accepted in

slowly formulated

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