CMUP Centro de Matemática Universidade do Porto

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

U.PORTO FACULDADE DE CIÊNCIAS UNIVERSIDADE DO PORTO

# Geometry of Algebraic Varieties
# PTDC/MAT-GEO/2823/2014
# Final Report

https://cmup.fc.up.pt/cmup/pbgothen/gav/

## 1 Main objectives and results of the project

The main research goals of this project were two-fold:

- To construct algebraic surfaces with specific numerical invariants both using algebro-geometric methods and as ball quotients, and to study their geometric properties.
- To improve the understanding of the geometry and topology of moduli spaces of $G$-Higgs bundles for a real reductive Lie group and to advance the understanding of Mirror Symmetry for Higgs bundle moduli.

The scientific objectives just stated were amply realized. Among the many significant results obtained, we highlight the following:

- The construction [25] of surfaces with canonical map of high degree, in particular of maximum degree for both regular and irregular cases.
- The construction [24] of the first example of a surface of general type with $\chi = 1$ and $K^2 = 8$ whose universal cover is not biholomorphic to the bidisk.
- Two works [19, 15]on mirror symmetry, the first one generalising the results of Hausel-Thaddeus to the parabolic setting, the second very recent one exhibiting pairs of BAA and BBB mirror branes which lie over the discrimant locus in the Hitchin base.
- The discovery [2, 3] of new "exotic" connected components in the moduli space of $G$-Higgs bundles for the group $G = \mathrm{SO}(p, q)$. This work throws a bridge between Higgs bundle theory and higher Teichmüller theory, and points the way forward to a better understanding for general real reductive Lie groups.

During the project a total of 15 papers in international peer reviewed journals were published or accepted for publication and a further 7 preprints have been published (almost all of which are submitted and currently under review). Additionally, 3 papers were published in conference proceedings (one of which contains significant original research) and one paper was published in a national journal. Also, numerous lectures were given by project members on their work in international conferences and seminars at research institutions worldwide. Additionally, an outreach event *Kolam* was organized in 2018.

We refer to the following sections for a complete description of the activities and scientific results of the project.

# 2 Detailed description of scientific results

## 2.1 Ball quotients

- C. Rito with Vincent Koziarz and Xavier Roulleau, (see [23]) proved that the orbifold ball quotient surface $S$, obtained by Martin Deraux [11] using a non-arithmetic lattice in PU(2,1), is the projective weighted plane $\mathbb{P}(1,3,8)$. Such a non-arithmetic group is quite rare and this was a major question left open by Deraux.

- C. Rito [25] showed the existence of a regular ball-quotient surface with canonical map of degree 36 and of an irregular ball-quotient surface with canonical map of degree 27, showing that the theoretical limits 36, 27 for regular, irregular surfaces, respectively, can indeed be attained. This was done using the Borisov-Keum equations of a fake projective plane and the Borisov-Yeung equations of the Cartwright-Steger surface.

- C.Rito, answering a question by Cartwright-Koziarz-Yeung [8], showed that the $\mathbb{Z}/3$-invariant fibres of the Albanese fibration of the Cartwright-Steger surface are smooth. This was done by computing the equations (over a finite field) of such fibres.

- C. Rito (in preparation with Xavier Roulleau), wrote an algorithm that, given matrices that generate a sublattice of PU(2,1), (i.e. the group of automorphisms of the unit ball in $\mathbb{C}^2$), computes a presentation for that group, as well as other properties, like listing torsion elements. This follows the techniques developed by Cartwright-Steger on the classification of the fake projective planes. Using the several new arithmetic and non-arithmetic lattices given by Deraux-Parker-Paupert (see [12], [13]), this algorithm will be used to study new ball quotient surfaces coming from these lattices.

## 2.2 Schoen surfaces

- C. Rito (with Xavier Roulleau and Alessandra Sarti, see [26]) gives an explicit construction of the family of Schoen surfaces, hitherto unknown, by computing equations for their canonical images, which are 40-nodal complete intersections of a quadric and the Igusa quartic in $\mathbb{P}^4$. A particularly interesting example, with 240 automorphisms and maximal Picard number, was also studied.

## 2.3 Coverings of surfaces with Kodaira dimension 0

- In work in preparation, C. Rito and E. Dias computed equations for the family of $\mathbb{Z}/2$-Godeaux surfaces $X$ studied by Stephen Coughlan [10]. More precisely, equations for the universal covering $Y$ (which is simply connected) with a fixed point free involution $\sigma$ such that $X = Y/\sigma$ is a Godeaux surface are given.

- Again in work in preparation together with Giancarlo Urzùa, they classify all possible degenerations from $\mathbb{Z}/2$-Godeaux surfaces to surfaces with a unique Wahl singularity. The only possibilities are that $X$ degenerates to a $D_{2,n}$ elliptic surface, with $n = 3, 4$ or 6, or $X$ degenerates to an Enriques surface, which are surfaces with Kodaira dimension 0. Examples are given covering all possibilities for the Enriques case, and $D_{2,n}$ with $n = 3, 4$.

- C. Rito (with Christian Gleissner and Roberto Pignatelli, see [17]) constructed an algorithm that, for a given value of the geometric genus $p_g$, computes all regular ($q = 0$) product-quotient surfaces with abelian group that have at most canonical singularities and have canonical system with at most isolated base points. This was used to show that there are exactly two families of such surfaces with canonical map of degree $d = K^2 = 32$. Also a construction of a surface with $q = 1$ and canonical map of degree $d = K^2 = 24$ is given. Previously only examples with $d \leq 24, q = 0$ and $d \leq 16, q > 0$ were known. The surfaces with $d = 32$ are coverings of Kummer surfaces, which are of Kodaira dimension 0.

- C. Rito (with Francesco Polizzi and Xavier Roulleau, see [24]) gives the first example of a surface of general type with $\chi = 1$ and $K^2 = 8$ whose universal cover is not biholomorphic to the bidisk. This is obtained as a double covering of an abelian surface, which is a surface with Kodaira dimension 0. The surface is rigid and unique up to complex conjugation. Apart from the family of product-quotient surfaces constructed by Penegini, this is the unique surface with these invariants and Albanese map of degree 2.

- M. Mendes Lopes (together with Abel Castorena and Gian Pietro Pirola, see [9]), proved by constructing explicit examples and giving a method of construction, the existence of fibrations of surfaces of general type over an elliptic curve having a unique singular fibre. This phenomenon was not known and believed to be impossible.

## 2.4 Connected components of Higgs bundle moduli

- *Maximal Higgs bundles for adjoint forms via Cayley correspondence.* For a fixed compact Riemann surface $X$, of genus at least 2, O. Garcia-Prada and A. Oliveira [16] counted the number of connected components of the moduli space of maximal Higgs bundles over $X$ for the Hermitian groups $\mathrm{PSp}(2n, \mathbb{R})$, $\mathrm{PSO}^*(2n)$, $\mathrm{PSO}_0(2, n)$ and $E_6^{-14}$. Hence the same result follows for the number of connected components of the moduli space of maximal representations of $\pi_1 X$ in these groups. We used the Cayley correspondence proved in paper by O. Biquard, O. García-Prada and R. Rubio [6] as our main tool.

- $\mathrm{SO}(p, q)$*-Higgs bundles and Higher Teichmüller components.* For many classes of Lie groups $G$, natural topological invariants completely label connected components of the moduli spaces of $G$-Higgs bundles on a compact Riemann surface $X$. Hitchin components in the split real form case, and maximal components in the Hermitian (tube type) case, were the only previously known cases where these invariants do not fully distinguish connected components. In addition, these components have great interest due to the fact that the representations of $\pi_1 X$ lying in it (under the non-abelian Hodge correspondence) have very special features, such as being discrete, faithful and having a certain positivity characteristic (such representations are called positive). For these reason, such components have been name higher Teichmüller components. In joint work [2, 3] with M. Aparicio-Arroyo, S. Bradlow, B. Collier and O. Garcia-Prada, P. Gothen and A. Oliveira have completed the proof of the existence of higher Teichmuller components in the moduli space of $\mathrm{SO}(p, q)$-Higgs bundles (for $p > 1$), this being the first case of a real Lie group

which is not split neither Hermitian, for which these components have been shown to exist. We obtained an explicit parametrization of these components via a 'Cayley correspondence' generalizing the usual parametrizations of both Hitchin and maximal components. In general, the $\mathrm{SO}(p,q)$ groups lie outside the above mentioned classes of real forms, but fit in a conjectural natural framework which seems to be associated to the existence of such components.

- *A general Cayley correspondence for higher Teichmüller components.* Following the previous work, P. Gothen and A. Oliveira, in collaboration with S. Bradlow, B. Collier, O. Garcia-Prada, started the abstract study on the existence of such components, for any real Lie group $G$. We deduced a complete, Lie theoretic, criterion for their existence. This is related to the existence of special nilpotents in the Lie algebra of $G$, which we introduced, and named "magical". Moreover, we found a complete parametrisation of these components, and shown that it recovers all the known cases so far, namely split, maximal Hermitian $\mathrm{SO}(p,q)$. We also proved that the representations in these components are positive. Finally, we proved that the Lie groups which admit magical nilpotents are exactly the same as the ones which admit positive representations (whose classification was obtained by O. Guichard and A. Wienhard [20]). A paper is in preparation. This work is related to task 4 of the application.

## 2.5 Mirror Symmetry for Higgs bundle moduli

- *Topological Mirror Symmetry for Parabolic Higgs bundles.* In 2003, T. Hausel and M. Thaddeus [21] proved that the Hitchin systems on the moduli spaces of $\mathrm{SL}(n,\mathbb{C})$- and $\mathrm{PGL}(n,\mathbb{C})$-Higgs bundles on a curve, verify the requirements to be considered mirror partners, as proposed by Strominger-Yau-Zaslow. According to the expectations coming from Physics, the generalized Hodge numbers of these moduli spaces should thus agree — this is the so-called topological mirror symmetry. Hausel and Thaddeus proved that this is the case for $n = 2, 3$ and gave strong indications that the same holds for any $n$ prime (and non-zero degree). Within the scope of this FCT project, P. Gothen and A. Oliveira [19] proved the version of topological mirror symmetry conjecture of Hausel–Thaddeus for the moduli space of strongly parabolic Higgs bundles, on a curve, of rank two and three, with full flags, for any generic weights. Although the main theorem is proved only for rank at most three, most of the results are proved for any prime rank.

- *Unramified covers and branes on the Hitchin system* P. Gothen and A. Oliveira in collaboration with E. Franco and A. Peon-Nieto [15] used the ideas of Hecke correspondence and fixed points under tensorization of finite order line bundles to explore mirror symmetry for the moduli space of Higgs bundles in the spirit of the seminal work of Kapustin and Witten [22]. More precisely, we exhibited pairs of dual branes for the Langlands self dual group $\mathrm{GL}(n,\mathbb{C})$. The interest of our construction relies on two aspects: firstly, as predicted by Kapustin and Witten, the branes we consider are sheaves (rather than just submanifolds) and the duality is realized via an explicit Fourier–Mukai transform; secondly, we made progress in the understanding of mirror symmetry in the singular locus of the Hitchin fibration, since the branes lie entirely over this locus. As far as we know, this is the first example of dual

branes lying over the singular locus where mirror symmetry is explicitly realized by a Fourier–Mukai transform.

## 2.6 Rigidity and flexibility for Higgs bundles

- P. Gothen in collaboration with S. Bradlow, O. Garcia-Prada and Jochen Heinloth [7] gave necessary and sufficient conditions for moduli spaces of semistable chains on a curve to be irreducible and non-empty. This gives information on the irreducible components of the nilpotent cone of $\mathrm{GL}_n$-Higgs bundles and the irreducible components of moduli of systems of Hodge bundles on curves. Moreover, As no co-primality restrictions are imposed, this can be applied to prove connectedness for moduli spaces of $\mathrm{U}(p,q)$-Higgs bundles, and also to show that the rigidity phenomenon for $\mathrm{U}(p,q)$-Higgs bundles can only happen in the maximal Toledo invariant case.

## 2.7 Quiver bundles

- P. Gothen and A. Nozad [18] have associated a new quiver bundle to the Hom-complex of two chains, and prove that stability of the chains implies stability of this new quiver bundle. Their approach uses the Hitchin-Kobayashi correspondence for quiver bundles and is used to give a new more conceptual proof of a key lemma on chains (due to Álvarez-Cónsul, García-Prada and Schmitt); this latter result has been important in the study of Higgs bundle moduli because holomorphic chains on a Riemann surface arise naturally as fixed points of the natural $\mathbb{C}*$-action on the moduli space of Higgs bundles.

## 2.8 Quiver varieties for symmetric quivers

- A. Araújo [1] introduced the concept of generalized quiver bundles for an arbitrary reductive group $G$. In the case when $G = \mathrm{O}(V)$ or $\mathrm{Sp}(V)$, these cab be interpreted as orthogonal (resp. symplectic) bundle representations of the symmetric quivers introduced by Derksen-Weyman. He also studied super-mixed quivers, which simultaneously involve both orthogonal and symplectic symmetries. In particular, he completely characterised the polystable such representations. He also established the Hitchin-Kobayashi correspondences for these objects, building on earlier work by other authors and taking into account the parameters in the moment map for the connections part.

## 2.9 Geometry of moduli spaces

- J. Silva worked with some techniques developed mainly in [4] to obtain some motivic information on the free abelian character varieties

$$\mathcal{M}_{\mathbb{Z}^r} G \;\cong\; \mathsf{hom}\left(\mathbb{Z}^r, G\right) /\!\!/ G$$

where $G = GL\left(n,\mathbb{C}\right)$, $SL\left(n,\mathbb{C}\right)$ or $Sp\left(n,\mathbb{C}\right)$. For the case $G = GL\left(n,\mathbb{C}\right)$, he managed to get formulas for their class in the Grothendieck ring of varieties $K\left(\mathcal{V}ar_{\mathbb{C}}\right)$ and so — namely through the Bittner isomorphism [5] — also in the Grothendieck

ring of Chow motives $K(\mathcal{M}ot_{\mathbb{Q}})$. These three quasi-projective varieties can be descbribed as a finite algebraic quotient. Using this description, he developed some strategies, using the main result in [4], to determine their class in the Grothendieck ring of the category of effective Chow motives endowed with an action of a finite group $K\left(\mathcal{M}ot_{\mathbb{Q}}^{F}\right)$. From this result, one can also recover the classes of $\mathcal{M}_{\mathbb{Z}^r}G$ in $K(\mathcal{M}ot_{\mathbb{Q}})$, in part through the results in [14].

- J. Silva made some simplifications in the joint work [14] with C. Florentino and extended the results from complex linear algebraic groups to include complex tori and real Lie groups.

# 3 FCT approved changes to the execution of the project

The research carried out followed broadly the timeline and tasks set out in the application. In particular the main scientific goals of the project were fully realised. However, as is customary in mathematical research, certain adaptations of the initially foreseen time line turned out to be necessary. The following specific changes occurred in relation to the original application:

- During the execution of the project it became clear that it was difficult to find candidates with the right profile for the initially planned initial research grants with the prior qualification of MSc (BI/mestre). On the other hand excellent candidates for initial research grants with the prior qualification of BSc (BI/licenciado) existed, so the type of the initial research grants was changed accordingly. Additionally, it turned out to be very useful to use the funding freed up by this change to add another postdoctoral grant (BPD) and one additional month of invited scientist grant (BCC), in order to attach to the project researchers with very specific and highly useful skills for the objectives of the project. These changes were authorized by the FCT (15/11/2016 and 13/08/2018).

- A small delay in the execution of certain project tasks occurred, on the one hand due to the hiring process of the two last project post-docs taking longer than expected, and on the other hand because of the need to develop new Lie theoretic methods in the study of $G$-Higgs bundle moduli. This led to the request for an extension to the project for 6 months, which was approved by the FCT (12/03/2019).

# 4 Advanced training

## 4.1 PhD theses

- *Representations of generalized quivers*, Ph.D. thesis, Artur de Araújo, Universidade do Porto, defended 2 June 2017, supervision Peter Gothen.

- *Surfaces of general type with non-birational canonical map*, Ph.D. thesis, Nguyen Bin, Universidade de Lisboa, defended 28 October 2019, supervision M. Mendes Lopes.

### 4.2 MSc theses

- *Representations on diagrams of categories and applications to parabolic bundles*, MSc thesis, Rui Prezado, Universidade do Porto, defended 17 July 2019, supervision Peter Gothena and André Oliveira.

### 4.3 Student introductory research projects

- Margarida Mendes Lopes supervised the 2nd year student Alexandre Bernardo in the programme Novos Talentos em Matemática (New talents in mathematics) of the Foundation Calouste Gulbenkian (academic year 2016-2017).
- Peter Gothen Supervised the 3rd year student João Rocha in the programme Novos Talentos em Matemática (New talents in mathematics) of the Foundation Calouste Gulbenkian (academic year 2016-2017).
- André Oliveira and Carlos Rito supervised Carlos André Vieira Macedo, who held a BI grant from the project in the period 01/11/2016-30/04/2017.
- Carlos Rito supervised Manuel José Ribeiro de Castro Silva Martins, who held a BI grant from the mathematics centre (CMUP) in the period 03/10/2016-02/01/2017.

## 5 Grants funded by the project

The following researchers were associated to the project as grant holders.

- André Macedo (BI 01/11/2016-30/04/2017)
- Artur Araújo (BI in the period 01/01/2017-31/03/2017)
- Eduardo Dias (BPD 01/09/2018-14/09/2019)
- Emilio Franco (BPD 01/02/2017-31/01/2018)
- Giancarlo Urzúa (BCC 01/02/2019-28/02/2019)
- Jaime Silva (BPD 01/11/2018-14/09/2019)
- Oscar Garcia-Prada (BCC 01/10/2016-31/10/2016, 01/04/2018-30/04/2018)
- Rui Prezado (BI 01/06/2018-31/05/2019)
- Xavier Roulleau (BCC 15/03/2019-14/04/2019)

## 6 Other activities

A very significant number[1] of missions and consultants were funded by the project. Even though electronic means of communications was used on a regular basis, face-to-face contact is necessary for collaborative mathematical research, and the participation in international conferences is an essential part of the activity of the team members, both

[1]The complete list of missions and consultants are available in the listings of the financial report.

to present the results of their work and to stay up to date with latest developments in the area. Additionally, two laptop computers and a hard disk were acquired at the start of the project for use by project members in the their relevant research.

# 7 Communication and Outreach

Additionally to the activities listed here, project members gave numerous seminars on their work at prestigious research institutions worldwide, such as the Hodge Institute (Edinburgh), Institute of Science and Technology (Vienna), and the Universities of Coimbra, Lisbon, Marseille, Maryland, Minho and Porto.

## 7.1 Lectures in Conferences

- Margarida Mendes Lopes: Searching for surfaces, Workshop Mathematics under construction, University of Potsdam, Germany, July 15, 2016.
- André Oliveira: Fixed point subvarieties, nilpotent cone and mixed Hodge polynomials of (parabolic) Higgs bundles, National Meeting of the Portuguese Mathematical Society, Barreiro, July 13, 2016.
- Carlos Rito: A surface with $\chi = 1$ and $K^2 = 8$ not covered by the bidisk, National Meeting of the Portuguese Mathematical Society, Barreiro July 13, 2016.
- Peter Gothen: Exotic connected components of $\mathrm{SO}(p, q)$-Higgs bundle moduli, Hitchin 70: Celebrating 30 years of Higgs bundles and 15 years of generalized geometry, ICMAT, Madrid, 12-16 September, 2016.
- André Oliveira: Topological mirror symmetry for parabolic Higgs bundles, Conference on Character Varieties and String Theory, Indian Institute of Science, Bangalore, 21 March 2017.
- Peter Gothen: Triples, chains and $\mathrm{U}(p, q)$-Higgs bundles, Geometry and Physics of Augmented Bundles: A celebration of Steve Bradlow's 60th birthday, Allerton (UIUC), Illinois, May 5-7, 2017
- André Oliveira: Topological mirror symmetry, fixed points and branes, Moduli Spaces: Conference in honour of S. Ramanan for his 80th birthday, ICMAT, Madrid, 6 November 2017.
- Peter Gothen: Exotic connected components in the moduli space of $\mathrm{SO}(p, q)$-Higgs bundles, Moduli Spaces: Conference in honour of S. Ramanan for his 80th birthday, ICMAT, Madrid, November 6-8, 2017.
- Carlos Rito:A surface with $p_g = q = 2$ and $K^2 = 8$ which is not uniformized by the bidisk, Seventh Iberoamerican Congress on Geometry, University of Valladolid, Spain, January 23, 2018.
- André Oliveira: Fibrados de Higgs e seus espaços moduli – Plenary Talk of the Encontro Nacional da Sociedade Portuguesa de Matemática (SPM), ESTG-IPBragança, Bragança, 11 July 2018.

- Carlos Rito: New surfaces with canonical map of high degree, National Meeting of the Portuguese Mathematical Society, Instituto Politécnico de Bragança, 2018/07/10.

- André Oliveira: Generalized Cayley correspondence and higher Teichmüller components for $\mathrm{SO}(p,q)$-Higgs bundles, Workshop on New trends in Higgs bundle theory, ICMAT, Madrid, 12–16 November 2018.

- André Oliveira: Extra components of Higgs bundles moduli spaces and generalized Cayley correspondence, Current Trends in Hitchin Systems, Buenos Aires, 17–21 December 2018.

- Carlos Rito: Surfaces with canonical map of maximum degree, Annual meeting of the Chilean Mathematical Society, Universidad de O'higgins, Chile, 2018/12/21.

- André Oliveira: Torsion line bundles and branes on the Hitchin system, Discussion Meeting on Bundles 2019, Tata Institute of Fundamental Research, Mumbai, India, 26 March 2019.

- Carlos Rito: Algebraic Surfaces and Computer Algebra, CMUP Post Doc Meeting III, University of Porto, 2019/05/17.

- Peter Gothen: Moduli of $G$-Higgs bundles and higher Teichmüller theory, Graduate Summer School on the Mathematics and Physics of Hitchin Systems, Simons Center for Geometry and Physics, Stony Brook, USA, May 27-31, 2019.

- Peter Gothen: Examples of higher Teichmüller components via $G$-Higgs bundles, Algebraic Analysis and Geometry with a view on Higgs bundles and $\mathcal{D}$-modules, Porto, June 3-7, 2019.

- Margarida Mendes Lopes: Surfaces and fibrations, Workshop "Algebraic surfaces and related topics", Kochi, Japan, 28/08/2019.

## 7.2 Organization of seminars and conferences

Seminars and conferences (co-)organized by project members on core topics of the project, not necessarily with project funding.

- Thematic session "Geometria algébrica" of the National Meeting of the Portuguese Mathematical Society, Barreiro July, 2016 (Margarida Mendes Lopes and Peter Gothen).

- Regular Geometry and Topology Seminar of CMUP (André Oliveira).

- Stacks and the Geometric Langlands Conjecture at the CMUP (working seminar March-May 2017, organised by Artur Araújo and Emilio Franco).

- Organization of the special session "Algebraic surfaces" of the Seventh Iberoamerican Congress on Geometry (Valladolid, 22-26 January 2018) (Margarida Mendes Lopes in collaboration with F. Monserrat).

- Hodge theory weekly seminar at CMUP (2018/2019, organized by Eduardo Dias and Jaime Silva).

- Special session "Geometria Algébrica" do Encontro Nacional da SPM, Bragança, dia 9 de Julho de 2018 (organised by Margarida Mendes Lopes and Peter Gothen).
- "Ball Quotient Surfaces and Lattices", 2019/02/25 -2019/03/01, Centre International de Rencontres Mathématiques, Luminy, France (Carlos Rito, Member of the Organising Committee).
- "Algebraic Analysis and Geometry with a view on Higgs bundles and $\mathcal{D}$-modules", Departamento de Matemática da Faculdade de Ciências da Universidade do Porto, 3-7 June 2019.
- "WM2 - Women in Mathematics Meeting", Universidade Nova de Lisboa, 22-24 July 2019 (Margarida Mendes Lopes, member of the scientific committee and organisation of the round table).
- Peter Gothen is a member of the VBAC (Vector Bundles on Algebraic Curves) committee, which serves as the scientific committee of the annual VBAC conferences.

### 7.3 Outreach

- Photography exhibition, *Kolam: An Ephemeral Women's Art Of South India* by Claudia Silva, Library of the Faculty of Science of the University of Porto. This was coupled with a special outreach event on April 13-15 with a recital and lectures by Claudia Silva, *Kolam: a women's tradition and its relation to education and ethnomathematics* and Oscar García-Prada, *Geometry of kolams*. Organized by Peter Gothen and Helena Mena-Matos.
- The article *On the construction of complex algebraic surfaces* by Carlos Rito explains for non-specialist mathematicians some of the central problems in the area and some of the recent results obtained by the author. Published in the Bulletin of CIM (Centro Internacional de Matemática) **38-39** (2017), 3-7.

## 8 Publications

The complete list of publications of the project with links to online versions of the papers can be found at the project web page `https://cmup.fc.up.pt/cmup/pbgothen/gav/`.

## References


[1] A. Araújo, *Generalized quivers, orthogonal and symplectic representations, and Hitchin-Kobayashi correspondences*, International Journal of Mathematics **30** (2019), no. 3, 1850085, 46 pp.

[2] M. Aparicio-Arroyo, S. Bradlow, B. Collier, O. Garcia-Prada, P. B. Gothen and A. Oliveira, $\mathrm{SO}(p,q)$*-Higgs bundles and higher Teichmüller components*, Inventiones Mathematicae, **218** (2019), 197-299.

[3] M. Aparicio-Arroyo, S. Bradlow, B. Collier, O. Garcia-Prada, P. B. Gothen and A. Oliveira, *Exotic components of* $\mathrm{SO}(p,q)$*-surface group representations, and their Higgs bundle avatars*, C. R. Acad. Sci. Paris, Ser. I **356** (2018), 666-673.

[4] S. del Baño Rollín and V. Navarro-Aznar, *On the motive of a quotient variety.* Collectanea Math. **49** (1998), 203–226.

[5] F. Bittner, *The universal Euler characteristic for varieties of characteristic zero*, Compos. Math. 140 (2004), no. **4**, 1011–1032.

[6] O. Biquard, O. García-Prada and R. Rubio, *Higgs bundles, the Toledo invariant and the Cayley correspondence* Journal of Topology **10** (2017), 795–826.

[7] S. B. Bradlow, O. Garcia-Prada, P. B. Gothen and J. Heinloth, *Irreducibility of moduli of semistable Chains and applications to U(p,q)-Higgs bundles*, Geometry and Physics: Volume 2, A Festschrift in honour of Nigel Hitchin, Oxford University Press, 2018, pp. 455-470.

[8] D. Cartwright, V. Koziarz, S.K. Yeung, *On the Cartwright-Steger surface,* J. Algebraic Geom. **26** (2017), no. 4, 655–689.

[9] A. Castorena, M. Mendes Lopes and G. P. Pirola, *Semistable fibrations over an elliptic curve with only one singular fibre*, Osaka J. Math **57** (2020), 9–15.

[10] S. Coughlan, *Extending hyperelliptic K3 surfaces, and Godeaux surfaces with* $\pi_1 = \mathbb{Z}/2$, J. Korean Math. Soc. **53** (2016), no. 4, 869–893.

[11] M. Deraux, *Non-arithmetic lattices from a configuration of elliptic curves in an Abelian surface*, Comment. Math. Helv. **93** (2018) 533–554.

[12] M. Deraux, J. Parker, J. Paupert, *New non-arithmetic complex hyperbolic lattices*, Invent. Math. **203** (2016), no. 3, 681–771.

[13] M. Deraux, J. Parker, J. Paupert, *New non-arithmetic complex hyperbolic lattices II,* arXiv:1611.00330 [math.GT].

[14] C. Florentino and J. Silva, *Hodge-Deligne polynomials of abelian character varieties*, Preprint arxiv:1711.07909.

[15] E. Franco, P. B. Gothen, A. Oliveira and A. Peon-Nieto, *Unramified covers and branes on the Hitchin system*, arXiv:1802.05237 [math.AG].

[16] O. Garcia-Prada and A. Oliveira, *Maximal Higgs bundles for adjoint forms via Cayley correspondence*, Geometriae Dedicata **190** (2017), 1-22.

[17] C. Gleissner, R. Pignatelli, C. Rito, *New surfaces with canonical map of high degree*, arXiv:1807.11854 [math.AG].

[18] P. B. Gothen and A. Nozad, *Quiver Bundles and Wall Crossing for Chains*, Geometriae Dedicata, **199** (2019), 137–146.

[19] P. B. Gothen and A. Oliveira, *Topological mirror symmetry for parabolic Higgs bundles*, Journal of Geometry and Physics, **137** (2019), 7–34.

[20] O. Guichard and A. Wienhard, *Topological invariants of Anosov representations*, Journal of Topology **3** (2010), 578–642.

[21] T. Hausel, M. Thaddeus, *Mirror symmetry, Langlands duality, and the Hitchin system*, Invent. Math. **153** (2003), 197–229.

[22] A. Kapustin, E. Witten, *Electric-magnetic duality and the geometric Langlands program*, Commun. Number Theory Phys. **1** (2007), 1–236.

[23] V. Koziarz, C. Rito and X. Roulleau, *The Bolza curve and some orbifold ball quotient surfaces*, Michigan Mathematical Journal (accepted).

[24] F. Polizzi, C. Rito, X. Roulleau, *A pair of rigid surfaces with $p_g = q = 2$ and $K^2 = 8$ whose universal cover is not the bidisk*, International Mathematics Research Notices, rny107 (published online: 05 June 2018).

[25] C. Rito, *Surfaces with canonical map of maximum degree*, Journal of Algebraic Geometry (accepted). arXiv:1903.03017 [math.AG].

[26] C. Rito, X. Roulleau, A. Sarti, *Explicit Schoen surfaces*, Algebraic Geometry **6** (2019), 410–426.

Porto, 20/01/2020

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