Higgs bundles, spectral data and applications

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The plan

1. Higgs bundles
   - The Hitchin fibration.
   - Spectral data approach.
   - Hyperkähler structure.
2. Real slices of Higgs bundles
   - A triple of involutions.
   - Branes and Langlands duality.
   - (B, A, A)-branes and low rank isogenies.
3. Monodromy of (B, A, A)-branes
   - (B, A, A)-branes and split real forms.
   - Braids and polyhedrons.
   - Character varieties.

Based on:
arXiv:1301.1981
arXiv:1111.2550

And work w/
D. Baraglia
1309.1195
1506.00372
& /w
S. Bradlow
1506.XXXXX

The Hitchin fibration

Consider a compact Riemann surface Σ of $g \geq 2$ and $K := T^*\Sigma$.

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E a holomorphic vector bundle, and
$\Phi : E \to E \otimes K$ holomorphic map.

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How can we construct families of them, and what can we say about their topology?

A triple of involutions $i_j : \mathcal{M}_{G_c} \to \mathcal{M}_{G_c}$

Defining families of branes in $\mathcal{M}_{G_c}$. Baraglia-S. 2013

G_c complex Lie group;

G real form of G_c fixed by an anti-holomorphic involution σ;

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$$i_1 : (E, \Phi) \mapsto (\sigma\rho(E), -\sigma\rho(\Phi))$$

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Fixes an (A, B, A)-brane, some of whose give representations that extend to a 3-manifold bounding Σ.

$i_3 := i_1 \circ i_2$ fixes an (A, A, B)-brane,“ pseudo real Higgs bundles”.

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Conjecture (Baraglia-S. 2013). Since $(A.B, A)$ are self dual, the dual brane for i_2 is the one fixed by ${}^Li_2$.

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The dual (B, B, B) branes for i_1 are supported over the moduli space of Higgs bundles for *Nadler's group*.

(B, A, A)-branes and low rank isogenies

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$$\mathcal{I}_3 : \mathcal{M}_{SL(4,\mathbb{C})} \to \mathcal{M}_{SO(6,\mathbb{C})}$$

$$\begin{tikzcd}
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The induced map is a 2^{2g+1} fold-cover $\mathcal{I}_2 : \mathcal{M}_{SL(2,\mathbb{R})\times SL(2,\mathbb{R})} \to \mathcal{M}_{SO(2,2)}$, of certain components in $\mathcal{M}_{SO(2,2)}$.

(B, A, A)-BRANES AND SPLIT REAL FORMS

CONSIDER G THE SPLIT REAL FORM OF G_c

Theorem (S., 2013)

The space $\mathcal{M}_G$ intersects the smooth fibres of the Hitchin fibration $h: \mathcal{M}_{G^c} \to \mathcal{A}_{G^c}$ in torsion two points.

For Example: the smooth fibres for $SL(n, \mathbb{C})$-Higgs bundles are $\mathrm{Prym}(S, \Sigma)$.

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Since we have a finite covering of the Hitchin base, we can study the monodromy action of loops in the base.

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for $SL(2, \mathbb{R})$-Higgs bundles and L-twisted rank 2 Higgs bundles

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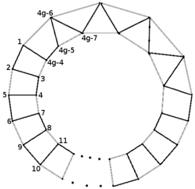

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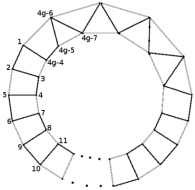

L-twisted Higgs bundles (Baraglia-S.'15). It is an affine moduli space - topology determined by the monodromty and the twisted Chern class.

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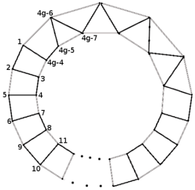

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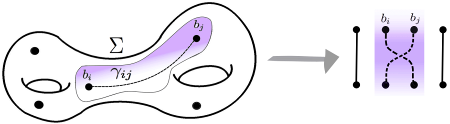

The monodromy action is given by Picard-Lefschetz transformations.

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Real character varieties $Rep(G) = Hom^{\text{red}}(\pi_1(\Sigma), G)/G$

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5. $3.2^{2g} + 2g - 4$ *for* $Rep_{2g-2}(Sp(4,\mathbb{R}))$;

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6. $6.2^{2g} + 4g^2 - 6g - 3$ *for* $Rep(SO(2, 2))$;

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(2) and (4) in [Goldman '88], (3) in [Xia '97, '99], (5) in [Gothen '01]

What is next?

Interface control and snow crystal growth

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(b) Department of Mathematics, University of Illinois, Urbana, IL 61801, USA

The growth of snow crystals is dependent on the temperature and saturation of the environment. In the case of dendrites, Reiter's local two-dimensional model provides a realistic approach to the study of dendrite growth. In this paper we obtain a new geometric rule that incorporates interface control, a basic mechanism of crystallization that is not taken into account in the original Reiter's model. By defining two new variables, growth latency and growth direction, our improved model gives a realistic model not only for dendrite but also for plate forms.

Keywords:

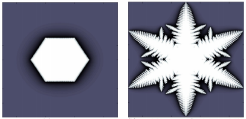
I. INTRODUCTION

Snowflake growth is a specific example of crystallization - how crystals grow and create complex structures. Because crystallization corresponds to a basic phase transition in physics, and crystals make up the foundation of several major industries, studying snowflake growth helps gaining understanding of how molecules condense to form crystals. This fundamental knowledge may help fabricate novel types of crystalline materials [7].

Whilst current computer modelling methods can generate snowflake images that successfully capture some basic features of real snowflakes, certain fundamental features of snowflakes growth are not well understood. One of the key challenges has been that the snowflake growth

Another approach in which snowflake growth is numerically simulated to produce images with mathematical models derived from physical principles is through computer modelling (e.g. see [3, 4, 15–17, 19]). By comparing computer generated images with actual snowflakes, one can correlate the mathematical models and their parameters with physical conditions. While computer modelling can generate snowflake images that successfully capture some basic features of actual snowflakes, so far there has been only limited analysis of these computer models in the literature.

In this paper we analyze snowflake growth simulated by the computer models so as to connect the microscopic and macroscopic views and to further our understanding of snowflake physics. The models that have been con-



(a) $\varepsilon = 0.1$ (b) $\varepsilon = 0.01,\ \alpha = 1,\ \beta = 0.4,\ \gamma = 0.001$

FIG. 14: Snowflake images generated by the enhanced Reiter's model with the new geometric rule.

Current trends on spectral data for Higgs bundles

September 26-27, 2015.

San Jose, California, USA.

San Francisco's view from Alcatraz.

Higgs bundles and their spectral data have had applications in different research areas of mathematics and physics. This weekend workshop is intended to bring young researchers in the area to highlight advances in their research, and inspire new perspectives from which to study Higgs bundles and its applications.

Invited Speakers:

- Lara Anderson, Virginia Tech, USA.
- Lucas Branco, University of Oxford, UK.
- Olivia Dumitrescu, Leibniz Universität Hannover, Germany.
- Laura Fredrickson, The University of Texas at Austin, USA.
- Victoria Hoskins, Freie Universität Berlin, Germany.
- Marina Logares, ICMAT, Spain.
- Alessia Mandini, PUC, Rio de Janeiro, Brazil.
- Ana Peón-Nieto, Ruprecht-Karls-Universität Heidelberg, Germany.

Spectral data for Higgs bundles

September 28 to October 2, 2015

at the

American Institute of Mathematics, San Jose, California

organized by

Joergen Ellegaard Andersen, David Baraglia, Philip Boalch, and Laura Schaposnik

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New perspectives on Higgs bundles, branes and quantization: June 13 – 17, 2016

Organized by: Lara B. Anderson, David Baraglia, Laura P. Schaposnik, and Vivek Shende

Dates: June 13 – 17, 2016

The aim of this workshop is to connect recent developments and open questions in the study of branes in the moduli space of Higgs bundles with other areas of mathematics and theoretical physics: differential and algebraic geometry, compact and non compact Calabi-Yau integrable systems, curve and brane quantization, and string vacuum spaces. In particular, one of the goals of the workshop is to bring to wider attention some of the problems in other parts of mathematics and physics that have arisen recently and which can be tackled from the perspective of Higgs bundles and spectral data.

Application deadline: April 13, 2016. Applicant will be notified soon after this date of their acceptance.

Gracias!