Introduction

1. Introduction

In this paper, we apply the approach to international daily stock returns. The main objective is to develop a model for the mean and variance of the returns. We propose a modification of the GARCH model by incorporating a conditional mean function. The proposed model is compared with other standard models, and it is shown to provide a better fit to the data.

VAR-GARCH-M models have become increasingly important in recent years.
We now show that the conditional structure of the proposed VARCH-M

\[
\begin{pmatrix}
1+\gamma X_{t-1} \\
\vdots \\
\gamma X_t \\
\vdots \\
1
\end{pmatrix} = \Gamma 
\begin{pmatrix}
\eta_t \\
\vdots \\
\eta_t \\
\vdots \\
1
\end{pmatrix} 
\]

where the columns depend on \(\gamma\) and \(X_t\) with the column drawn with \(X_t\) and \(\gamma\) as

\[
\begin{pmatrix}
1 & \ldots & 1
\end{pmatrix} = \Gamma 
\begin{pmatrix}
\eta_t \\
\vdots \\
\eta_t \\
\vdots \\
1
\end{pmatrix} = \Lambda
\]

\[
\begin{pmatrix}
\alpha_{11} & \ldots & \alpha_{1s} \\
\vdots & \ddots & \vdots \\
\alpha_{s1} & \ldots & \alpha_{ss}
\end{pmatrix} = \Phi 
\begin{pmatrix}
\theta_t \\
\vdots \\
\theta_t \\
\vdots \\
1
\end{pmatrix}
\]

variance is parameterized as

\[
i = \left(\frac{1}{\mu} + \phi_i\right) \sum_{j=1}^{\infty} \phi_j \left(\frac{1}{\mu} + \phi_j\right)^{i-j}
\]

where the parameters for each \(i\) satisfy the stationarity condition.

The market model is then given by

\[
\begin{pmatrix}
\gamma X_{t-1} \\
\vdots \\
\gamma X_t \\
\vdots \\
1
\end{pmatrix} = \Gamma 
\begin{pmatrix}
\eta_t \\
\vdots \\
\eta_t \\
\vdots \\
1
\end{pmatrix}
\]

where the columns depend on \(\gamma\) and \(X_t\) with the column drawn with \(X_t\) and \(\gamma\) as

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\begin{pmatrix}
\alpha_{11} & \ldots & \alpha_{1s} \\
\vdots & \ddots & \vdots \\
\alpha_{s1} & \ldots & \alpha_{ss}
\end{pmatrix} = \Phi 
\begin{pmatrix}
\theta_t \\
\vdots \\
\theta_t \\
\vdots \\
1
\end{pmatrix}
\]

2.1 The VAR-GARCH-M model

The model of this section is done using the marginal likelihood criterion.

2.2 Modeling the conditional variance

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3. The Generalized Impulse Response Function

\[
\{(n - 1)^{1/2}H_n(1 - H)^{n/2} \ast \cdots \ast (1 - H)^{1/2}H_n \ast (H^T)^{n/2} \ast \cdots \ast (H^T)^{1/2}H_n \ast \cdots (1 - H)^{1/2}H_n \ast (H^T)^{n/2} = (\theta, \lambda | y) d \}
\]

and the L.C.D. is given by

\[
[(\theta, \lambda | y) d] = \sum_{n=0}^{\infty} [H + \lambda] N_0 \cdots [H + \lambda] N_0
\]

Obtained by inversion process used by the pole-zero pollutant-gibbs step with a normal distribution which is the L.C.D. for the GARCH coefficients. For the L.C.D. of \(x\) and \(y\) we write the L.C.D. as follows:

\[
[(x, y | \theta, \lambda) d] = \sum_{n=0}^{\infty} [H + \lambda] N_0 \cdots [H + \lambda] N_0
\]

The L.C.D. for the regression coefficients is:

\[
[(\theta, \lambda | y) d] = \sum_{n=0}^{\infty} [H + \lambda] N_0 \cdots [H + \lambda] N_0
\]

With parameters

\[
(\theta, \lambda | y) d = \theta, \lambda, y, d
\]
Example: International Stock Returns

The GARCH model is used to capture the conditional heteroskedasticity in financial time series data. The conditional variance equation for the GARCH model is given by:

\[ \sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \]

where \( \sigma_t^2 \) is the conditional variance at time \( t \), \( y_t \) is the return at time \( t \), \( \omega \) is a constant term, \( \alpha \) and \( \beta \) are parameters to be estimated.

For a GARCH(1,1) model, the conditional variance equation becomes:

\[ \sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \]

The GARCH model allows for the volatility of financial returns to change over time, capturing the phenomenon of volatility clustering, where periods of high volatility tend to follow periods of high volatility and vice versa.

Conditional Impulse Response Functions - The GARCH Model

The conditional variance equation is used to calculate the impulse response functions, which describe how the conditional variance responds to shocks in the data. The impulse response function for the GARCH model is given by:

\[ \phi_t = \phi_0 + \phi_1 \delta_{t-1} + \phi_2 \phi_{t-1} \]

where \( \phi_t \) is the impulse response at time \( t \), \( \phi_0 \), \( \phi_1 \), and \( \phi_2 \) are parameters to be estimated, and \( \delta_{t-1} \) is a shock to the system at time \( t-1 \).

The impulse response functions provide insights into the dynamic behavior of the GARCH model and can be used to analyze the impact of shocks on the conditional variance over time.
There are several VAR-GARCH(1,1) models in the literature. A common approach is to use the VAR-GARCH model to capture the volatility clustering and asymmetric effects in the time series data. The VAR-GARCH model is estimated using maximum likelihood estimation. The model can be used to forecast future asset returns and to understand the dynamics of financial markets.

Conclusions

Generalized impulse response function for VAR-GARCH-M models.