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Generalized Impulse Response Functions for VAR-GARCH-M Models

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Abstract. VAR-GARCH-M models have become increasingly important for estimating volatility returns and exogenous shocks for international finance data. Based on the Bayesian VAR-GARCH-M approach of Polasek and Ren (1999) we propose a new concept of generalized impulse response function based on a posterior sample of an MCMC estimation method. The proposal is an extension of the Koop et al. (1996) approach and can be calculated for shocks in the mean and variances of the time series. We apply this approach to international daily stock returns from June 21^{st}, 1996 to June 22^{nd}, 1998.

1 Introduction

Various methods have been recently applied to explore the international financial markets by econometric volatility models. In this paper we extend the estimation approach of Polasek and Ren (1999) to analyse the transmission of shocks in a country AR-GARCH-M model.

For the estimation approach we have chosen a Bayesian MCMC (Markov Chain Monte Carlo) method since reliable methods for the likelihood estimation of the VARCH-M model seem to be difficult to obtain in closed form. Furthermore, the MCMC approach allows the introduction of new concepts and to find exact (small sample) results for characteristics of the dynamic process, like the impulse response function or the predictive distributions.

In section 2 we introduce the basic VAR-GARCH-M model and in section 3 we present the estimation results. We show how the Gibbs sampler and the Metropolis step for the ARCH parameters is implemented in the simulation using the full conditional distributions. The lag orders of the model are estimated by the marginal likelihoods criterion (see Pelloni and Polasek 1998). The time series are checked for stationarity using the fractional marginal likelihood approach as in Polasek and Ren (1998). Since the VAR-GARCH-M model is a nonlinear multivariate model we have to extend the concept of the impulse response function to mean and volatility response. This is done using the concept of Koop et al. (1996) by defining the impulse response function as a numerical derivative for the s-step ahead forecast with respect to a unit shock. In a similar way we define impulse response functions for the conditional variances. A previous classical approach can be found in Lin (1997).

The posterior mean of the estimated coefficients shows that there is a rich interaction pattern between the coefficients of the mean equation and the volatility equations. The ARCH-M coefficients exhibit a substantial reaction to volatilities and all the impulse response function have a quick decay. The predictive distributions are compared to the usual VAR approach and they show considerable improvements. Section 2 introduces the VAR-GARCH-M model and section 3 the generalized impulse response function. Section 4 describes the 3-dimensional model on international stock returns and in a final section we conclude our approach.

2 Modeling and estimation

The modeling of financial time series has been enriched by the class of ARCH-in-mean or ARCH-M processes which were introduced by Engle, Lilien and Robins (1987). The following section describes the extension of ARCH-in-mean models to multivariate VAR-GARCH-M processes from a Bayesian point of view. The models are estimated by MCMC methods and model selection is done using the marginal likelihood criterion.

2.1 The VAR-GARCH-M model

To describe the interactions of returns and conditional variances in a VAR model we extend the univariate ARCH-M model of Engle et al. (1987) to the multivariate case. Thus, we define a VAR(k) model of dimension M, i.e. the VAR(k)-GARCH(p,q)-M(r) model, in the following way:

$$y_t^l = \beta_0^l + \sum_{m=1}^{M}\sum_{i=1}^{k} \beta_i^{lm} y_{t-i}^m + \sum_{m=1}^{M}\sum_{i=1}^{r} \psi_i^{lm} h_{t-i}^m + u_t^l \qquad (1)$$

with heteroskedastic errors $u_t^l \sim N[0, h_t^l]$, $l = 1, \ldots, M$. The conditional variance is parameterized as

$$h_t^l = \alpha_0^l + \sum_{m=1}^{M}(\sum_{i=1}^{p} \alpha_i^{lm} h_{t-i}^m + \sum_{i=1}^{q} \phi_i^{lm} u_{m,t-i}^2), \qquad (2)$$

where the parameters for each l satisfy the stationarity condition

$$\sum_{m=1}^{M}(\sum_{i=1}^{p} \alpha^{lm} + \sum_{i=1}^{q} \phi_i^{lm}) < 1, \qquad (3)$$

with all coefficients being positive: $\alpha_0^{lm} > 0$, $\alpha_i^{lm} \geq 0$, $\phi_i^{lm} \geq 0$ and $m, l = 1, \ldots, M$. Equation (1) can be written as

$$\mathbf{y}_t = \beta_0 + \sum_{i=1}^{k} \beta_i \mathbf{y}_{t-i} + \sum_{i=1}^{r} \Psi_i vech\mathbf{H}_{t-i} + \mathbf{u}_t = \mu_t + \mathbf{u}_t, \qquad (4)$$

where $\mathbf{y}_t = (y_{t1}, \ldots, y_{tM})'$ is an $M \times 1$ vector of observed time series at time t, β_i ($i = 1, \ldots, k$) and Ψ_i ($i = 1, \ldots, r$) are fixed $M \times M$ coefficient matrices, $\beta_0 = (\beta_{10}, \ldots, \beta_{M0})'$ is a fixed $M \times 1$ vector of intercept terms, $\mu_t = (\mu_t^1, \ldots, \mu_t^M)'$ is the $M \times 1$ vector of conditional means and $\mathbf{u}_t = (u_{t1}, \ldots, u_{tM})^T$ is an $M \times 1$ vector of error terms. $\mathbf{H}_t$ is the conditional covariance matrix of the M dimensional observation at time t and $vech\mathbf{H}_t$ is the vectorization of the lower half of the covariance matrix.

The above model is rewritten as a multivariate regression system

$$\mathbf{Y} = \mathbf{B}\mathbf{X} + \Psi\tilde{\mathbf{H}} + \mathbf{U}, \qquad (5)$$

with $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_T]_{(M\times T)}$ and $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_T]_{(M\times T)}$, where the coefficient matrices are defined as

$$\mathbf{B} = [\beta_0, \beta_1, \ldots, \beta_k]_{(M\times(\tilde{M}k+1))}, \quad \Psi = [\Psi_1, \ldots, \Psi_r]_{(M\times\tilde{M}r)}.$$

The regressor matrices are partitioned in transposed form as

$$\mathbf{X} = [\mathbf{x}_0, \ldots, \mathbf{x}_{T-1}]_{((1+\tilde{M}k)\times T)}, \quad \tilde{\mathbf{H}} = [\tilde{\mathbf{h}}_0, \ldots, \tilde{\mathbf{h}}_{T-1}]_{(\tilde{M}r\times T)}$$

with the columns defined with $\tilde{M} = M(M+1)/2$ as

$$\mathbf{x}_t = \begin{pmatrix} 1 \\ \mathbf{y}_t \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{y}_{t-k+1} \end{pmatrix}, \quad \tilde{\mathbf{h}}_t = \begin{pmatrix} vech\mathbf{H}_t \\ \cdot \\ \cdot \\ \cdot \\ vech\mathbf{H}_{t-r+1} \end{pmatrix}.$$

We now show that the conditional structure of the proposed VARCH-M model makes the MCMC and the Gibbs sampler convenient to apply in blocks of the parameters.

The Bayesian VAR(k)-GARCH(p,q)-M(r) model is then given by

$$\mathbf{Y} \sim N_{T\times M}[\mathbf{B}\mathbf{X} + \Psi\tilde{\mathbf{H}}, diag(\mathbf{H}_1, \ldots, \mathbf{H}_T)], \qquad (6)$$

$$vech\mathbf{H}_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i vech(\mathbf{u}_{t-i}\mathbf{u}'_{t-i}) + \sum_{j=1}^{p} \Phi_j vech\mathbf{H}_{t-j},$$

and the prior distributions are chosen from the families of normal distributions, hence

$$\mathbf{B} \sim N_{M\times(1+\tilde{M}k)}[\mathbf{B}_*, \Sigma_{B_*} \otimes \mathbf{I}_M], \qquad (7)$$

$$\Psi \sim N_{M\times\tilde{M}r}[\Psi_*, \Sigma_{\Psi_*} \otimes \mathbf{I}_M],$$

where all of the hyper-parameters (which are denoted with a star) are known a priori. The joint distribution for the data $\mathbf{Y}$ and the parameters $\theta = (\mathbf{B}, \Psi, \mathbf{A}, \Phi)$is with $\mathbf{A} = (\alpha_0, \alpha_1, \ldots, \alpha_q)$ and $\Phi = (\phi_0, \phi_1, \ldots, \phi_p)$

$$\begin{aligned} p(\theta, \mathbf{Y}) = & N[\mathbf{Y}|\mathbf{BX} + \Psi\tilde{\mathbf{H}}, diag(\mathbf{H}_1, \ldots, \mathbf{H}_T)] \\ & \cdot N[\mathbf{B}|\mathbf{B}_*, \Sigma_{B_*} \otimes \mathbf{I}_M] \cdot N[\Psi|\Psi_*, \Sigma_{\Psi_*} \otimes \mathbf{I}_M] \\ & \cdot \prod_{i=0}^{p} N_0^{\infty}[\alpha_i|\alpha_i^*, \Sigma_{\alpha i}] \cdot \prod_{i=1}^{q} N_0^{\infty}[\Phi_i|\Phi_i^*, \Sigma_{\Phi i}]. \end{aligned} \tag{8}$$

As prior distribution for the GARCH coefficients we use the positive truncated normal distribution (N_0^{∞}) since the variance components of the GARCH equation showed be positive. For the VAR regression coefficients we use the "tightness prior" of Litterman (1986) since the GARCH coefficients have to be positive for the prior means we assume $\alpha_* = \mathbf{0.01I_{1+p+q}}$ and for the VAR coefficients $\mathbf{B}_* = \mathbf{0}$, $\Psi_* = \mathbf{0}$ and for the prior precision matrices we assume the following diagonal tightness structure $\Sigma_{\mathbf{B}*}^{-1} = diag(\varepsilon, 1, \ldots, k)$, $\Sigma_{\Psi*}^{-1} = diag(1, \ldots, r)$, $\Sigma_{\alpha i}^{-1} = i\mathbf{I}_{\tilde{M}}$, $\Sigma_{\phi i}^{-1} = i\mathbf{I}_{\tilde{M}}$, and for the inverse variance of the intercepts we choose ε to be a small number like 10^{-6}.

2.2 The full conditional distributions (f.c.d.)

This section derives the full conditional distributions (f.c.d.) for the MCMC sampling simulation process. To simplify notation for the f.c.d. of the parameters we introduce the following notation for a partitioned matrix. If $\mathbf{H} = diag(\mathbf{H}_1, \ldots, \mathbf{H}_T)$ is a $TM \times TM$, $\mathbf{W}$ a $r \times T$, and $\mathbf{V}$ a $T \times k$ matrix, then we define the special matrix

$$\begin{aligned} < \mathbf{w}_t\mathbf{H}_t\mathbf{v}_t >_{rM\times kM} &= (\mathbf{W} \otimes \mathbf{I}_M) diag(\mathbf{H}_1, \ldots, \mathbf{H}_T)(\mathbf{V} \otimes \mathbf{I}_M) \\ &= \begin{pmatrix} \sum_t \mathbf{w}_{1t}\mathbf{H}_t\mathbf{v}_{t1}, \ldots, \sum_t \mathbf{w}_{1t}\mathbf{H}_t\mathbf{v}_{tk} \\ \ldots \\ \sum_t \mathbf{w}_{rt}\mathbf{H}_t\mathbf{v}_{t1}, \ldots, \sum_t \mathbf{w}_{rt}\mathbf{H}_t\mathbf{v}_{tk} \end{pmatrix}. \end{aligned}$$

The f.c.d. for the regression coefficients B. The full conditional density for $\mathbf{B}$ is a multivariate normal distribution

$$p(\mathbf{B}|\mathbf{Y}, \theta^c) = N_{M\times(1+\tilde{M}k)}[\mathbf{B}_{**}, \mathbf{D}_{B_{**}}], \tag{9}$$

with the parameters

$$\mathbf{D}_{B_{**}}^{-1} = \mathbf{I}_M \otimes \Sigma_{B_*}^{-1} + < \mathbf{x}_t'\mathbf{H}_t^{-1}\mathbf{x}_t >,$$

$$\mathbf{B}_{**} = \mathbf{D}_{B_{**}}[vec(\Sigma_{B_*}\mathbf{B}_* + < \mathbf{x}_t'\mathbf{H}_t^{-1}\tilde{\mathbf{y}}_t >)],$$

where $\tilde{\mathbf{y}}_t$ is the t^{th} row of $\tilde{\mathbf{Y}} = \mathbf{Y} - \Psi\tilde{\mathbf{H}}$ and $\theta^c = (\Psi, \mathbf{A}, \Phi)$ denotes a vector of all parameters save the arguments of the full conditional distribution.

The f.c.d. for the regression coefficients Ψ. The f.c.d. is given by

$$p(\Psi|\mathbf{Y}, \theta^c) = N_{M\times\tilde{M}r}[\Psi_{**}, \mathbf{D}_{\Psi_{**}}] \tag{10}$$

with

$$\mathbf{D}_{\Psi_{**}}^{-1} = \mathbf{I}_M \otimes \Sigma_{\Psi_*}^{-1} + < \mathbf{x}_t'\mathbf{H}_t^{-1}\mathbf{x}_t >,$$

$$\Psi_{**} = \mathbf{D}_{\Psi_{**}}[vec(\Sigma_{\Psi_*}\Psi_* + < \mathbf{x}_t'\mathbf{H}_t^{-1}\tilde{\mathbf{y}}_t >)]$$

and $\tilde{\mathbf{y}}_t$ is the t^{th} row of $\tilde{\mathbf{Y}} = \mathbf{Y} - \mathbf{BX}$.

Note that the Gibbs sampling steps in step **a)** and **b)** can be combined if there is enough computational capacities available.

The f.c.d. for the GARCH coefficients. For the f.c.d. of α_i and ϕ_i we use the Metropolis-within-Gibbs step with a normal distribution which is obtained by an iteration proposal given by

$$vec\alpha_i \sim N[vec\hat{\alpha}_i, \hat{\Sigma}_{\alpha_i}],$$

$$vec\Phi_i \sim N[vec\hat{\Phi}_i, \hat{\Sigma}_{\Phi_i}],$$

and the f.c.d. is given by

$$p(\alpha, \Phi|\mathbf{Y}, \theta^c) = \prod_{t=1}^{T} N[\mathbf{y}_t|\mu_t, \mathbf{H}_t] \tag{11}$$

with μ_t given in (4) and the normal distribution being proportional to

$$N[\mathbf{y}_t|\mu_t, \mathbf{H}_t] \propto |\mathbf{H}_t|^{-1/2} exp\{-\frac{1}{2}(\mathbf{y}_t - \mu_t)'\mathbf{H}_t^{-1}(\mathbf{y}_t - \mu_t)\}.$$

3 The generalized impulse response function

Impulse response function are used in VAR systems to describe the dynamic behaviour of the time series system with respect to unit shocks in the residuals of the time series. For non-linear time series systems like multivariate GARCH models the concept has to be extended to generalized impulse response function.

Based on the approach of Koop et al. (1996) we propose the following definition for the generalized impulse response function for the VAR-GARCH-M model

$$IMP_{\mu}(s, u_t, \Omega_{t-1}) = \frac{1}{s}[E_t(\mathbf{y}_{t+s}|u_t, \Omega_{t-1}) - E_t(\mathbf{y}_{t+s}|\Omega_{t-1})], \tag{12}$$

where Ω_t is the information set up to time t and u_t is an arbitrary current shock. We will use unit-vectors or 1 standard deviation shocks (with different signs) for exploring the dynamic behavior of the estimated system. The expectation is taken as the mean of the predictive distribution and is estimated by the average over the simulated future paths of the MCMC output.

The estimates of the s-step future mean observations at time t $\hat{\mathbf{y}}_{t+s} = E(\mathbf{y}_{t+s}|\ldots)$ are given by

$$\hat{\mathbf{y}}_{t+s} = \frac{1}{M}\sum_{m=1}^{M}\left[\mathbf{B}_0^{(m)} + \sum_{i=1}^{k}\mathbf{B}_i^{(m)}E_t\hat{\mathbf{y}}_{t+s-i} + \sum_{i=1}^{r}\Psi_i^{(m)}vech\mathbf{H}_{t+s-i}^{(m)}\right], \quad (13)$$

where the conditional expectation is given by

$$E_t\mathbf{y}_{t+s} = \begin{cases} \mathbf{y}_{t+s} & \text{for } s \le 0, \\ \hat{\mathbf{y}}_{t+s} & \text{for } s > 0, \end{cases} \quad (14)$$

and the variance equation is also calculated recursively with conditional means for the residual, i.e.

$$vech\mathbf{H}_{t+s}^{(m)} = \mathbf{A}_0^{(m)} + \sum_{i=1}^{q}\mathbf{A}_i^{(m)}vechE_t(\mathbf{u}_{t+s-i}\mathbf{u}_{t+s-i}') + \sum_{j=1}^{p}\Phi_j^{(m)}vech\mathbf{H}_{t+s-j}^{(m)}. \quad (15)$$

The conditional expectation of the residuals at time t is defined by

$$E_t\mathbf{u}_{t+s} = \begin{cases} \hat{\mathbf{u}}_{t+s} & \text{for } s \le 0, \\ 0 & \text{for } s > 0, \end{cases} \quad (16)$$

where $\mathbf{u}_{t+s}$ are the residuals of the m-th simulation of the MCMC output. The conditional mean for the "additive shock" conditional expectation is given by

$$E_t(\mathbf{u}_{t+s}^{(m)}|\mathbf{u}_{t+1}, \Omega_t) = \begin{cases} \hat{\mathbf{u}}_{t+s}^{(m)} & \text{for } s \le 0, \\ \mathbf{u}_j & \text{for } s = 0 \\ 0 & \text{for } s > 1, \end{cases} \quad (17)$$

where $\mathbf{u}_j$ could be the j-th unity vector ($\mathbf{e}_j$ of dimension M) or scaled by one standard error. A negative shock $\mathbf{u}_j = -\mathbf{e}_j$ could also be used as could any other interesting design of shocks.

In the same line we can define the generalized impulse response function for the volatilities

$$IMP_\Sigma(s, \mathbf{u}_t, \Omega_{t-1}) = \frac{1}{s}[E_t(\mathbf{H}_{t+s}|\mathbf{u}_t, \Omega_{t-1}) - E_t(\mathbf{H}_{t+s}|\Omega_{t-1})].$$

As before, the estimates of the future volatility matrices are given by the conditional expectation at time t, i.e. $\hat{\mathbf{H}}_{t+s} = E_t(\mathbf{H}_{t+s}|\ldots)$ and are calculated

from the MCMC output as

$$vech\hat{\mathbf{H}}_{t+s}^{(m)} = \frac{1}{M}\sum_{m=1}^{M}[\alpha_0^{(m)} + \sum_{i=1}^{q}\alpha_i^{(m)}vechE_t(\mathbf{u}_{t+s-i}\mathbf{u}_{t+s-i}') + \sum_{j=1}^{p}\Phi_j^{(m)}vech\mathbf{H}_{t+s-j}^{(m)}],$$

where the $\mathbf{u}_t$ are the base line shocks in (16) or the additive shock in (17).

In particular we are interested in the impulse responses of the main diagonal of $\mathbf{H}_t$, which are the variances $h_{t,11}, \ldots, h_{t,MM}$ with respect to a squared shock

$$IMP(s, \mathbf{u}_{tj}^2, \Omega_{t-1}) = \frac{1}{s}[E_t(h_{jj,t+s}|\mathbf{u}_t, \Omega_{t-1}) - E_t(h_{jj,t+s}|\Omega_{t-1})],$$

e.g.,

$$IMP(h_{t+s}|i \to j) = \frac{1}{s}[\hat{h}_{jj,t+s}(u_{it+1}^2 = 1) - \hat{h}_{jj,t+s}(u_{it+1}^2 = 0)] = \frac{1}{s}[\hat{h}_{i\to j}^{(1)} - \hat{h}_{i\to j}^{(0)}],$$

where $\hat{h}_{i\to j}^{(1)} = \hat{h}_{jj}(s|u_{i,t+1}^2 = 1)$ is the j-th diagonal element of $\hat{\mathbf{H}}_{t+s}$ if the additive impulse is set to $u_{i,t+1} = 1$ in the i-th component and $\hat{h}_{i\to j}^{(0)}$ is the diagonal element of $\hat{\mathbf{H}}_{t+s}$ if the base line shocks are used. $\hat{\mathbf{H}}_{t+s}$ is the mean of the MCMC forecast sample. Standard deviations of the impulse response function can be estimated by calculating the standard deviations of the MCMC forecast sample (and the above formulas).

4 Example: International stock returns

We have estimated a 3 dimensional VAR-GARCH-M model for the Nikkei, the DAX and the Dow Jones stock returns, daily data from June 21^{st}, 1996 to June 22^{nd}, 1998. We have tested for a break point and found Oct. 23, 1997 to be one (see Polasek and Ren 1999). The marginal likelihoods are calculated from the MCMC output by the method of Chib and Jeliazkov (1999). The marginal likelihood is the nominator and the denominator of a Bayes factor and can be described as the "mean value" of the likelihood function after the parameters are integrated out with an informative prior distribution $p(\theta)$

$$ml_1 = \int p(y|\theta_1)p(\theta_1)d\theta_1,$$

ml_1 denotes the likelihood of the model and $p(y|\theta_1)$ is the conditional likelihood function and θ_1 are the parameters of the (first) model. The values of

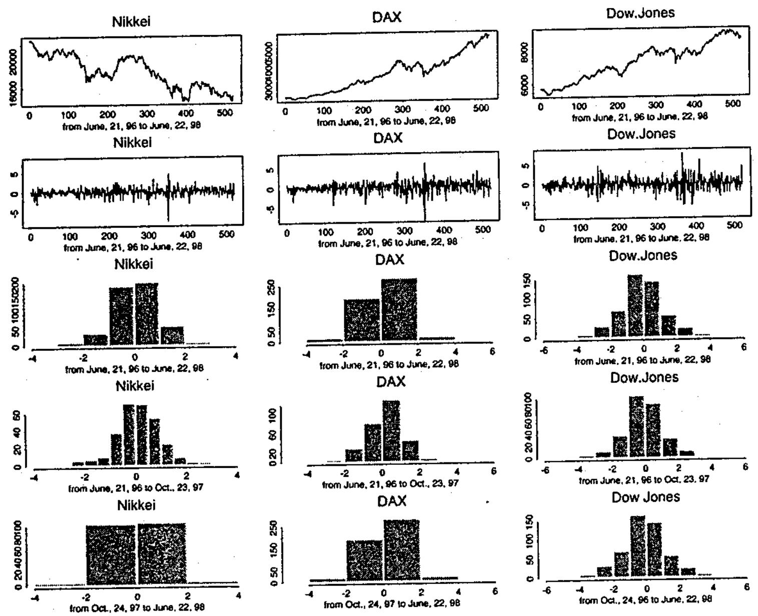

Fig. 1. Stock indices of Japan, Germany and USA (Nikkei, DAX and Dow Jones) from June, 21^{st}, 1996 to June, 22^{nd}, 1998, first row: daily data, second row: first differences of logs.

				total	period 1	period 2
k	r	p	q	06.21.96-06.22.98	06.21.96-10.23.97	10.24.97-06.22.98
1	0	0	0	-3417.2881	-1932.6271	-1305.7182
1	1	1	1	-3276.5365	-1894.6250	-1112.5263
1	2	1	1	-3140.5312	-1441.6236	-1273.9072
1	1	2	1	-2571.9042	-1632.7321	-1128.6527
1	1	1	2	-2469.5412	-1638.5326	-1038.2091
1	1	2	2	-2249.6109*	-1397.7273	-1226.9042
2	0	0	0	-3511.7826	-1987.1281	-1321.7281
2	1	2	2	-2636.4545	-1497.6242	-1077.4233
2	1	1	1	-2844.3320	-1354.6271	-1005.1167*
2	1	2	1	-3122.5321	-1232.6477	-1025.6277
2	2	2	2	-3368.7743	-1155.7272*	-1263.7273

Table 1. The log marginal likelihoods (ML) for the VAR(k)-GARCH(p,q)-M(r) model (for y_t^1 = Nikkei, y_t^2 = DAX, y_t^3 = Dow Jones)

the marginal likelihoods for the different order of the VAR-GARCH-M model can be found in Table 1.

For the total period the VAR(1)-GARCH(2,2)-M(1) is the best while for the first period before the Asian crisis the VAR(2)-GARCH(2,2)-M(2) model turns out to be the best while after Oct. 24, 1997 it is the VAR(2)-GARCH(1,1)-M(1) model.

The impulse response functions are shown in Figure 2 and 3 for the whole period, while Figures 8 and 4 show the pre-Asian-crisis period and Figures 6 and 7 for the period after Oct. 24, 1997.

	VAR-GARCH-M		VAR	
	mean	Std. error	mean	Std. error
Nikkei	-0.00029	0.00085	0.00373	0.00147
DAX	0.00139	0.00201	0.00089	0.00501
Dow Jones	-0.00049	0.00099	-0.00149	0.00149

Table 2. The mean and standard error of the one step ahead forecast period for stock indices with the VAR(1)-GARCH(2,2)-M(1) and the VAR(2) models

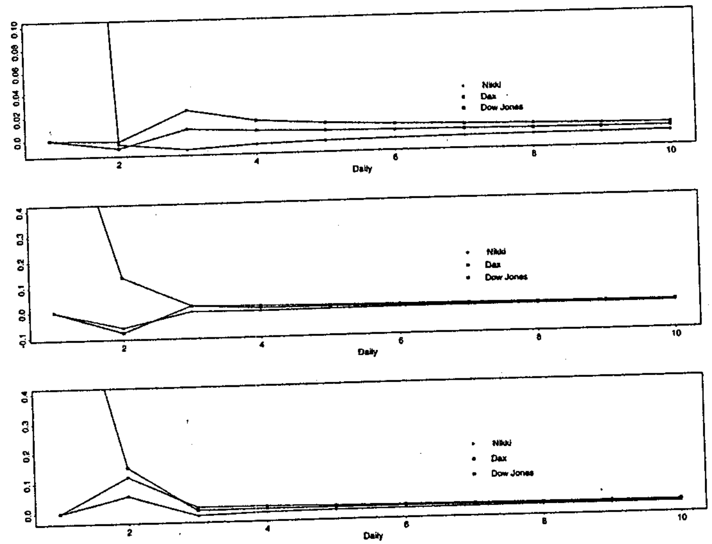

Fig. 2. Impulse response plots (for means) of stock returns for the VAR(1)-GARCH(2,2)-M(1) model: unit impulses for Nikkei, DAX and Dow Jones

The biggest change can be found for the volatility: Because dynamic interactions between volatilities are more active in the period before the Asia crisis, the impulse response have been reacting longer to shocks in the period before than in the period after Oct. 24, 1997. Interestingly, the DAX volatilities in the first period than the other two stock returns. Except for the Nikkei response, the impulse response functions of the mean returns are unaffected by the Asia crisis and very short lived in the period before and after Oct. 24, 1997.

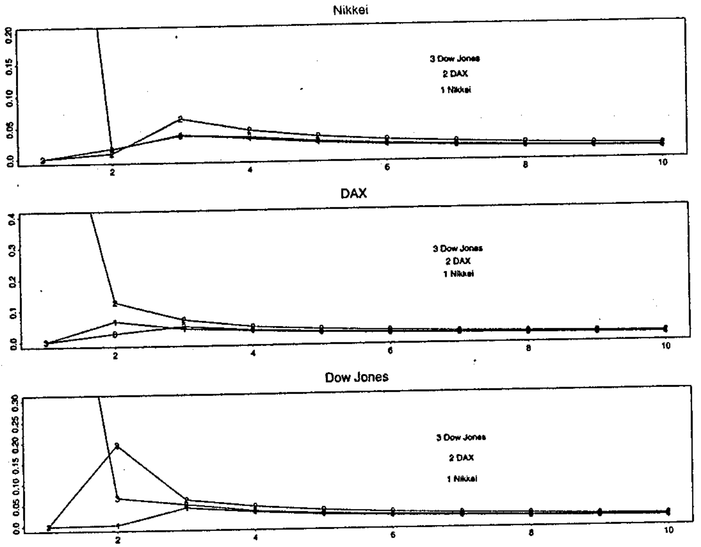

Fig. 3. Impulse response function (for the volatilities) of stock returns of the VAR(1)-GARCH(2,2)-M(1) model: unit impulses for Nikkei, DAX and Dow Jones

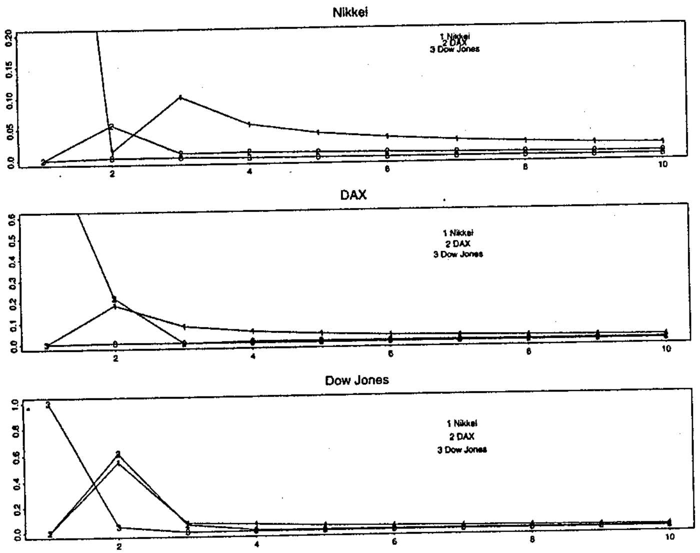

Fig. 4. Impulse response plots (for means) of stock returns for the VAR(1)-GARCH(2,2)-M(1) model: unit impulses for Nikkei, DAX and Dow Jones from 06.21.96 to 10.23.97

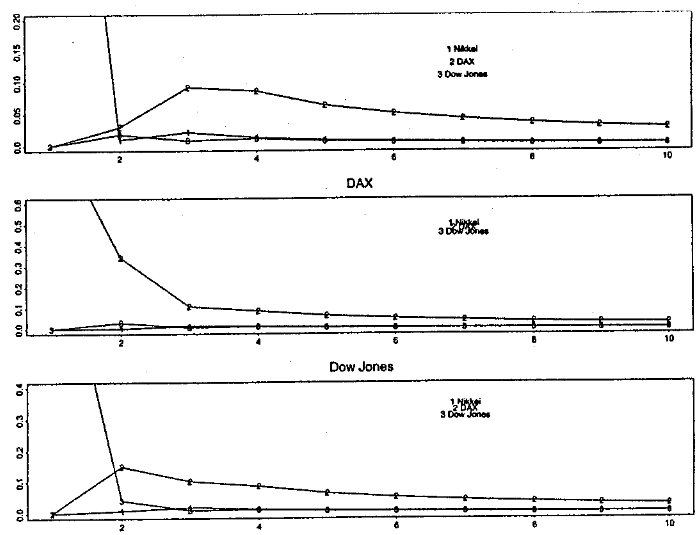

Fig. 5. Impulse response function (for the volatilities) of stock returns of the VAR(1)-GARCH(2,2)-M(1) model: unit impulses for Nikkei, DAX and Dow Jones from 06.21.96 to 10.23.97

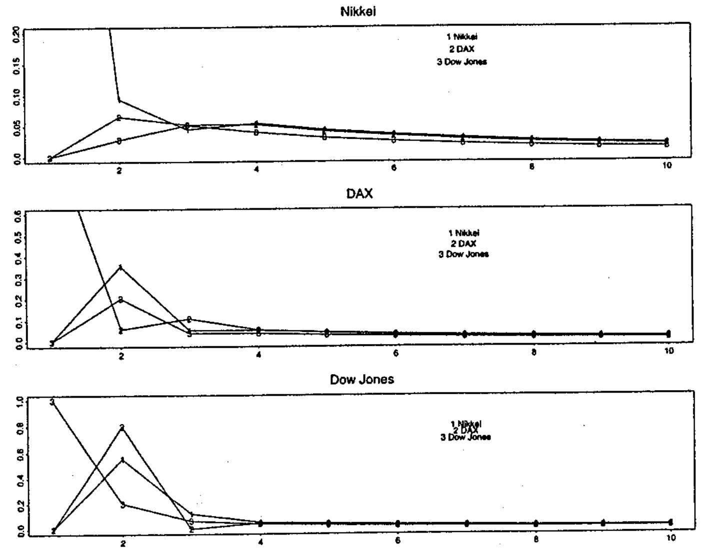

Fig. 6. Impulse response plots (for means) of stock returns for the VAR(1)-GARCH(2,2)-M(1) model: unit impulses for Nikkei, DAX and Dow Jo nes from 10.24.97 to 06.22.98

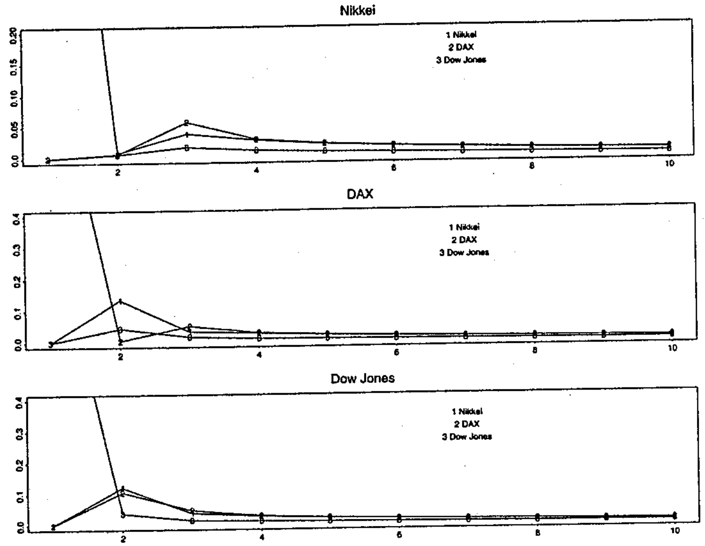

Fig. 7. Impulse response function (for the volatilities) of stock returns of the VAR(1)-GARCH(2,2)-M(1) model: unit impulses for Nikkei, DAX and Dow Jon es from 10.24.97 to 06.22.98

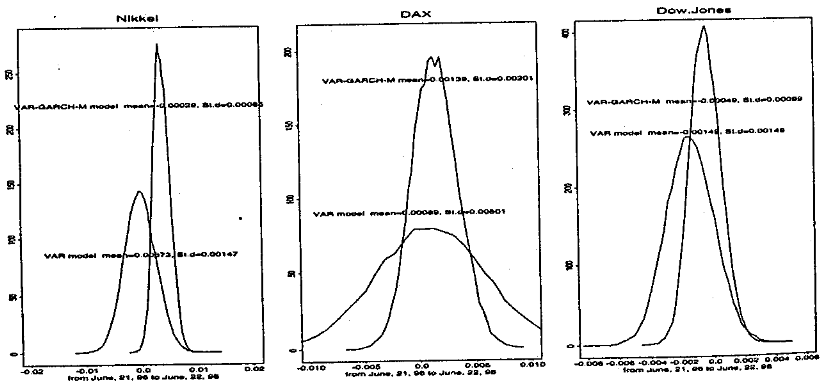

Fig. 8. Comparison of predictive density of stock index returns for the VAR(1)-GARCH(2,2)-M(1) and VAR(2) model

Finally we have compared the one step ahead predictive densities for the simple VAR model with the VAR-GARCH-M model in Figure 8. We see that the information gain in the VAR-GARCH-M model is reflected in smaller variances of the predictive density.

5 Conclusions

In Polasek and Ren (1999) a 3-dimensional model for stocks returns in the US, Germany and Japan was estimated by MCMC methods and tested for structural breaks. It was found that Oct. 24^{th}, 1997 was a break point for the 3 time series. A new concept of impulse response functions was proposed for this type of non-linear multivariate time series models, which is evaluated for MCMC outputs. The results are extended to impulse responses of the volatilities and we have compared the impulse response function of the VAR-GARCH-M model before and after the break point. Because of smaller lag interactions, the impulse response die off after the break point factor than before the "Asia" break point. The one step ahead prediction of the VAR-GARCH-M model shows a smaller variance and is also better in terms of the MSE (further details on forecast comparisons for volatile time series can be found in Polasek 1999).

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