Dynamical Systems Seminar

Date. February 02, 14h30

Place. Room M031

Speaker. Fernando Moreira[1] (CMUP – FCUP)

Title. Recurrence along non-polynomial sequences using ultrafilters

Abstract. Let (X, μ) be a probability measure space and $T : X \to X$ a μ-preserving map. The classical Poincaré recurrence theorem states that for any measurable $A \subset X$ with $\mu(A) > 0$, the corresponding return times

$$R_A = \{n \in \mathbb{N} : \mu(A \cap T^{-n}A) > 0\}$$

is an infinite set of integers.

Over the years it was revealed, by a lot of results, that the set R_A has quite intricate combinatorial and number-theoretical properties. One of these results is the *Khintchine's recurrence theorem* stating that, for every $\varepsilon > 0$ and any polynomial map with $p(0) = 0$ and $p(\mathbb{N}) \subset \mathbb{N}$, the set of return times

$$R_{\varepsilon,q,A} = \left\{n \in \mathbb{N} : \mu(A \cap T^{-p(n)}A) > \mu^2(A) - \varepsilon\right\}$$

is a syndetic one (i.e. has uniformly bounded gaps).

In this seminar we present an extension of this theorem to the case of non-polynomial maps. Its proof is based on the use of an important family of finitely additive probabilities on $\mathbb{N}$, the so-called ultrafilters.

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