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UC|UP MATH PhD Program

Research Seminar Program
UC|UP Joint PhD Program in Mathematics

Date and Time. October 27, 2015 - 15h00

Place. Room 5.5, Department of Mathematics, University of Coimbra

Speaker. Saeid Alirezazadeh [1]

Title. On Pseudovarieties of Forest Algebras

Abstract. Forests algebras are used in the theory of formal languages. They consist of two monoids, the horizontal one H and the vertical one V, with an action of V on H, and a complementary axiom of faithfulness. The main example is the forests algebras of plane forests and contexts, that is to say plane forests with a deleted leaf, which is the free object of the theory. In the study of forest algebras one of the main difficulty is how to handle the faithfulness property.

A pseudovariety is a class of finite algebras of a given signature, closed under the taking of homomorphic images, subalgebras and finitary direct products.

A profinite algebra is defined to be a projective limit of a projective system of finite algebras.

We tried to adapt in this context some of the results in the theory of semigroups, specially the results on relatively free profinite semigroups which have been shown to be an important tool in the study of pseudovarieties of semigroups.

We recall definition of forest algebras and state several basic results concerning definition and properties of forest algebras and the free forest algebra which are used later on.

Bojańczyk and Walukiewicz in ["Forest algebras", Logic and Automata, 2008, pp 107-131] defined the syntactic forest algebra over a forest language. We define a new version of syntactic congruence of a subset of the free forest algebra, not just a forest language, which is used in the proof of an analog of Hunter's Lemma ["Certain finitely generated compact zero-dimensional semigroups", 1988, pp 265-270]. The new version of syntactic congruence is the natural extension of the syntactic congruence for monoids in case of forest algebras. We show that for an inverse zero action subset and a forest language which is the intersection of the inverse zero action subset with the horizontal monoid, the two versions of syntactic congruences coincide. Almeida in ["Profinite semigroups and applications", Structural Theory of Automata, Semigroups, and Universal Algebra, 2005, pp 1-45] established some results on metric semigroups. We adapted some of his results to the context of forest algebras. We define on the free forest algebra a pseudo-ultrametric associated with a pseudovariety of forest algebras. We show that the analog of Hunter's Lemma holds for metric forest algebras, which leads to the result that zero-dimensional compact metric forest algebras are residually finite. We show an analog of Reiterman's Theorem ["The Birkhoff theorem for finite algebras", 1982, pp 1-10], which is based on a study of the structure profinite forest algebras.

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[1]Saeid Alirezazadeh is a former student for the Joint PhD Program in Mathematics UC|UP, recently defended his PhD title under the supervision of Prof. Jorge Almeida, and working in the area of "Semigroups, Automata and Languages".

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