Seminar on Representation Theory and Related Areas: 6th Workshop
Univ. Algarve, September 2nd, 2017

Split Lie-Rinehart algebras

Helena Albuquerque

In this talk we will speak about split Lie-Rinehart algebras. Lie-Rinehart algebras were introduced by J. Herz, being their theory mainly developed by R. Palais and G. Rinehart. A Lie-Rinehart algebra can be thought as a Lie K-algebra, which is simultaneously an A-module, where A is an associative and commutative K-algebra, in such a way that both structures are related in an appropriate way. In the last years, Lie-Rinehart algebras have been considered in many areas of Mathematics, particularly from a geometric viewpoint and of course from an algebraic viewpoint.

Moreover, we recall that the class of split Lie algebras is specially related to the addition of quantum numbers, graded contractions and deformations. For instance, for a physical system L, it is interesting to know in detail the structure of the split decomposition because its roots can be seen as certain eigenvalues which are the additive quantum numbers characterizing the state of such system. We note that determining the structure of different types of split algebras is becoming more meaningful in the area of research of Mathematical Physics.

CMUC & Departamento de Matemática
Universidade de Coimbra, Portugal
lena@mat.uc.pt

Thick subcategories, arc-collections and mutation

Nathan Broomhead

I will explain some work, in which I describe the lattices of thick subcategories of discrete derived categories. This is done using certain generating collections of exceptional and sphere-like objects related to non-crossing configurations of arcs in a geometric model.

School of Computing, Electronics and Mathematics
Faculty of Science & Engineering
Plymouth University, UK
nathan.broomhead@plymouth.ac.uk

Negative Calabi-Yau triangulated categories

Raquel Simões

Calabi-Yau (CY) triangulated categories are those satisfying a useful and important duality, characterised by a number called the CY dimension. Much work has been carried out on understanding positive CY triangulated categories, especially in the context of cluster-tilting theory. Even though CY dimension is usually considered to be a positive (or fractional) number, there are natural examples of CY triangulated categories where this “dimension” or parameter is negative, for example, stable module categories of self-injective algebras. Therefore, negative CY triangulated categories constitute a class of categories that warrant further systematic study.

In this talk, we give a brief introduction to this theory, and give special focus to a Calabi-Yau reduction theorem for this class of categories, giving a powerful inductive technique for the construction of systems of generators.

FCT, University of Lisbon, Portugal
rcoelhosimoes@campus.ul.pt

Some invariants of the super Jordan plane

Andrea Solotar

Hochschild cohomology and its Gerstenhaber algebra structure are relevant invariants: they are invariant by Morita equivalences, by tilting processes and by derived equivalences.

The computation of these invariants requires a resolution of the algebra considered as a bimodule over itself. Of course, there is always a canonical resolution available, the bar resolution, very useful from a theoretical point of view, but not very satisfactory in practice: the complexity of this resolution rarely allows explicit calculations to be carried out.

Nichols algebras are generalizations of symmetric algebras in the context of braided tensor categories. These are graded algebras, which first appeared in an article by Nichols in 1978, in which the author looked for examples of Hopf algebras. They are fundamental objects for the classification of pointed Hopf algebras, as was shown by the work of Andruskiewitsch and Schneider.

Heckenberger classified finite-dimensional Nichols algebras of diagonal type up to isomorphism. The classification separates the Nichols algebras in different classes: Nichols algebras of the Cartan type, essentially related to the finite quantum groups of Lusztig; Nichols algebras related to finite quantum supergroups and a third class related to contragradient Lie superalgebras. Later, Angiono described the definition relations of the Nichols algebras of the list of Heckenberger.

The problem of finite generation of the chomology of a Hopf algera is re- lated to Hochschild cohomology, since for any augmented algebra, the former graded space is isomorphic to a direct summmand of the latter.

In a joint work with Sebastián Reca, we computed the Hochschild ho- mology and cohomology of $\mathsf{A} = k\langle x, y \mid x^2, y^2x - xy^2 - xyx\rangle$, called the super Jordan plane, when $\operatorname{char}(k) = 0$ and k is algebraically closed. This is the Nichols algebra $\mathcal{B}(\mathsf{V}(-1, 2))$, whose Gelfand-Kirillov dimension is 2. The main results we obtained are the following.

- We give explicit bases for the Hochschild homology and cohomology spaces.
- We describe the cup product. From this description we see that the isomorphism between $\mathrm{H}^{2p}(\mathsf{A}, \mathsf{A})$ and $\mathrm{H}^{2p+2}(\mathsf{A}, \mathsf{A})$, where $p > 0$, is given by the multiplication with an element of $\mathrm{H}^2(\mathsf{A}, \mathsf{A})$, and similarly for the odd degrees.
- We describe Lie algebra structure of $\mathrm{H}^1(\mathsf{A}, \mathsf{A})$, which turns out to be isomorphic to a Lie subalgebra of the Virasoro algebra.

- We describe the $H^1(\mathsf{A},\mathsf{A})$-module structure of the vector spaces $H^n(\mathsf{A},\mathsf{A})$ for all $n \geq 2$ and classify these representations.

Departamento de Matemática, Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires, Argentina
& IMAS, UBA-CONICET, Argentina
asolotar@dm.uba.ar

Categorical representations of dihedral groups

Daniel Tubbenhauer[1]

This talk is meant to be an example-based introduction to 2-representation theory (in the sense of Chuang–Rouquier, Khovanov–Lauda and Mazorchuk–Miemietz).

I will focus on one (completely explicit) example, i.e. the categorical representation theory of dihedral groups. For these groups already several new phenomena appear–most of which are invisible in classical representation theory–and which might lead to interesting connections and applications in the years to come.

[1] Joint work with Ben Elias, Marco Mackaay, Volodymyr Mazorchuk and Vanessa Miemietz.

Mathematisches Institut
Universität Bonn, Germany
dtubben@math.uni-bonn.de

Borel-Schur algebra and the 0-Hecke monoid

Ivan Yudin[1]

First we determine a coherent presentation of the 0-Hecke monoid H and then we construct an action of H on the category of modules over the Borel-Schur algebra.

[1] Joint work with Ana Paula Santana.

CMUC & Departamento de Matemática
Universidade de Coimbra, Portugal
yudin@mat.uc.pt