

# Quadratic decomposition of Appell polynomial sequences

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## Abstract

We proceed to the quadratic decomposition of an Appell sequence and we prove that the four derived sequences obtained by this approach are also Appell sequences with respect to another (lowering) operator, which we call as  $\mathcal{F}_\varepsilon$ , where either  $\varepsilon = 1$  or  $\varepsilon = -1$ . Thus, we introduce and develop the concept of the Appell polynomial sequences with respect to the operator  $\mathcal{F}_\varepsilon$  (where  $\varepsilon$  is a parameter belonging to the field of the complex numbers): the  $\mathcal{F}_\varepsilon$ -Appell sequences. The orthogonal polynomial sequences that are also  $\mathcal{F}_\varepsilon$ -Appell correspond to the Laguerre sequences with parameter  $\varepsilon/2$ . Indeed, this brings an entirely new characterisation of the Laguerre sequences.

*Key words:* orthogonal polynomials, Appell sequences, classical polynomials  
MSC 2000: 33C45, 42C05

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<sup>1</sup> Work partially supported by the Centro de Matemática da Universidade do Porto, financed by FCT (Portugal) through the programmes POCTI (Programa Operacional “Ciência e Tecnologia e Inovação”) and POSI (Programa Operacional Sociedade da Informação), with national and European Community structural funds.

The first author would also like to thank to FCT (Portugal) by the support gave through POCI 2010 (Programa Operacional Ciência e Inovação 2010), with national and European Community structural funds.