A note on Pérez de Moya's *Principios de Geometria* (1584)

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ABSTRACT. In 1584, the Spanish mathematician Juan Pérez de Moya published a geometric work strongly practical in character, titled *Principios de geometria, de que se pod[r]an aprovechar los estudiosos de artes liberales, y todo hombre que su officio le necessitare a tomar la regla y co[m]pas en la mano. Con el medir, y dividir tierras*. A copy of it is now kept in the National Library of Lisbon and, as far as we know, no Spanish public library holds a copy of it. So far, no secondary bibliography has taken notice of this work. In this article we briefly describe the *Principios de Geometria*, pointing to the main differences with Moya's other geometric texts.

RE SUME. En 1584, le mathématicien espagnol Juan Pérez de Moya a écrit un travail géométrique dont le principal l'objectif était servir comme un outil pour les différents métiers. Le volume a le titre *Principios de geometria, de que se pod[r]an aprovechar los estudiosos de artes liberales, y todo hombre que su officio le necessitare a tomar la regla y co[m]pas en la mano. Con el medir, y dividir tierras* et il est conservé dans la Bibliothèque National de Lisbonne. À notre avis, aucune bibliothèque publique Espagnole possède une copie de cette œuvre de Moya. En conséquence, le texte dont ont vient de parler n'a jamais pris l'attention qu'il mérite. Dans cet article nous présentons une bref description des *Principios de Geometria*, en nous arrêtant tout particulièrement sur ses différences avec d'autres travaux géométriques de l'auteur.

Introduction

As is well known, Juan Pérez de Moya (Santisteban del Puerto, 1513 – Granada, 1596) was one of the most popular sixteenth-century mathematical authors in the Spanish Monarchy, then a vast and loose aggregate of American newly conquered territories and

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European principalities and kingdoms. A Roman Catholic priest and a competent mathematics teacher, Pérez de Moya addressed his many writings mostly to a public of mathematical practitioners[1]. Besides his mathematical writings, he also published in 1585 an interesting *Philosophia secreta: donde debaxo de historias fabulosas, se contiene mucha doctrina, provechosa a todos los estudios: con el origen de los idolos o dioses de la gentilidad*, a work that draws inspiring moral lessons from classical sources and mythology. His best known and most influential work doubtless was his *Arithmetica practica y especulativa*, first published in 1562 and countless times reprinted through the mid eighteenth century. The National Library of Lisbon keeps a work by Pérez de Moya titled *Principios de geometria, de que se pod[r]an aprovechar los estudiosos de artes liberales, y todo hombre que su officio le necessitare a tomar la regla y co[m]pas en la mano. Con el medir, y dividir tierras*[2]. Printed in Madrid in 1584 by Francisco Sánchez, it features Moya's authorship in the title page and is dedicated to Juan Baptista Gentil[3]. It is a little octavo volume, bound in parchment, with 127 sheets (numbered on the front page only) of which the first is missing. It ends with a non-numbered sheet reading: "Impresso en Madrid en casa de Francisco Sanchez. Ano de mil y quinientos y ocheta y quatro" (Printed in Madrid, in the house of Francisco Sanchez. Year of one thousand and five hundred eighty four). What makes this little book interesting is the fact that apparently no Spanish public library holds a copy of it, nor does it appear in the Catálogo Colectivo del Patrimonio Bibliográfico

[1] On Pérez de Moya, see [Leal y Leal 1972], [Clavería 1995] and [Valladares Reguero 1997].

[2] The book belongs to the Biblioteca Nacional de Lisboa and is catalogued as Res. 6553P. It is referenced in the catalogue *O Livro Científico dos Séculos XV e XVI, Ciências Físico-Matemáticas na Biblioteca Nacional* [Leitão e Martins 2004]. This catalogue was printed following the exhibition *Conta, Peso e Medida: A Ordem Matemática e a Descrição Física do Mundo*, opened to the public in the National Library of Lisbon Biblioteca from 2 December 2004 through 5 March 2005.

[3] Juan Baptista Gentil was a patron of Pérez de Moya, defraying the publication costs of his work *Philosophia secreta: donde debaxo de histórias fabulosas se contiene mucha doctrina, provechosa a todos los estudios: con el origin de los idolos o dioses de la gentilidad*, published in 1585. Pérez de Moya also devoted this work to him, and claimed that he would dedicate him anyother text that he might write.

Español (CCPBE)[4], nor is it mentioned in any bio-bibliography of Pérez de Moya examined by us — excepted the very recent one edited by V. Navarro *et al*[5]. Classical sources like [Fernández de Navarrete 1851], [Picatoste y Rodríguez 1891], [Domínguez Berrueta 1899], and López Piñero's *Diccionario histórico de la ciencia moderna en España*, as well as the recent and careful articles by [Leal y Leal 1972], [Clavería 1995], and [Valladares Reguero 1997], which have so much contributed to clarify Pérez de Moya's life and social context, contain no reference to Moya's *Principios de Geometria*. As a consequence, there is no secondary literature on this hitherto unnoticed work by Moya. More generally, it is also true that Moya's geometry has scarcely received any attention so far. In what follows, we present a brief description of the *Principios de Geometria* along with a few words devoted to Pérez de Moya's approach to geometry in his other works.

Pérez de Moya's geometry

Pérez de Moya first dealt with geometrical topics in the *Libro quarto* of his *Arithmetica practica y speculatiua*, published in Salamanca in 1562[6]. In keeping with the general orientation of this work, geometry was presented in three short chapters of an uilitary character under the title "Trata algunas reglas de Geometria pratica necessarias para el medir de las heredades". Essentially, they contain some results related to land

[4] The on-line Catálogo Colectivo del Patrimonio Bibliográfico Español is found at http://www.mcu.es/ccpb/index.html.

[5] [Navarro Brotons, Salavert Fabiani, Rossello Botey, Darás Roman 1999]. The book was already referenced in PORBASE, the union catalogue of Portuguese libraries, which can be consulted on-line at http://www.porbase.org.

[6] [Pérez de Moya 1562].

measuring. With little modifications, these topics reappear in the many successive editions of Pérez de Moya's influential *Arithmetica practica y speculatiua*[7].

The brief extension devoted to geometry in the *Arithmetica practica y speculatiua* should not mislead us as to the general interest that Pérez de Moya had for the subject[8]. In fact, the quantity and variety of information given in these few pages suggests that he might have had already in mind the composition of a more comprehensive account of geometrical results, such as the one he offered a few years later. In 1568 he published the *Obra intitulada Fragmentos Mathematicos*[9], entirely devoted to geometrical and astronomical topics. This is a two-volume work: the first, dated from 1568, is titled *Libro Primero que trata de Geometria Practica*; the second, printed in 1567, *Trata de cosas de astronomia, y geographia, y philosophia natural, y sphere, y astrolabio, y navegacion, y reloges*. Pérez de Moya used both volumes as authoritative sources of reference in his other works, including later editions of his popular *Arithmetica practica*[10].

Pérez de Moya's ambitious goal in writing this treatise was to provide a comprehensive encyclopeadic treatement of all the mathematics needed in the liberal arts. Yet, according to Moya's preface addressed to the reader, difficulties related to the cost of printing had not allowed him to complete his plan:

> Acontescido me há en esta obra (benévolo y prudente Lector) lo q̃ dizen acontescio al que se quito el sayo para poder saltar mas, y salto menos.

[7] Keeping account only of editions whose year of printing is known, there are four editions in the 16th century, eleven editions in the 17th, and ten in the 18th (plus the modern, critical 20th - century edition).

[8] Notice that in the 1998 edition of the *Aritmética prática y* speculativa [Consolación Baranda 1998] this chapter occupies only 12 of the 618 pages of the treatise.

[9] *Obra intitulada Fragmentos Matemáticos, en que se tratan cosas de Geometria y Astronomia, y Geographia, y Philosophia natural, y Sphera, y Astrolabio, y Nauegacion, y Reloxes*. Ordenada por Iuan Perez de Moya. Libro primeiro - Geometría Práctica,1568; Libro segundo - trata de cosas de astronomia y geographia y philosophia natural y sphera y astrolabio y navegación y reloges, 1567.

[10] See, for example [Pérez de Moya 1569] where Moya says: "Si dixere bolas en lugar de quadrad, lee cubicar. Lee sobre esta Geometria el tractado que intitulamos Fragmentos mathematicos".

> Digo esto, porq̃ quando determine començar la, tenia animo de no dexar materia tocãte a las artes liberales de q̃ cũplidamẽte no tratasse. Mas quando cõsidere la costa, y vários moldes q̃ eran menester, acorte el camiño, y cõpuse una obrezilla intitulada Fragmentos Mathematicos (...)[11].

This "little" book (Pérez de Moya calls it *obrezilla*, "tiny work") has about 380 sheets, including a 47-sheet index[12]. In the final form Moya gave it, the work deals with

> cosas de Geometria pratica, y Astronomia, y Geographia, y Sphera, y Astrolabio, y Navegaciõ, y Reloxes, y otras cosas a este proposito q̃ se platican, y cõmunican ordinariamẽte[13].

For our purposes, the most important chapters are those dealing with geometry and planimetry, included in the first volume. We briefly resume here their main contents. Pérez de Moya dedicates 40 chapters to geometry proper. He studies there the triangle (definition and construction of triangles) and the circle (how to construct the diameter and the centre of a circle); the determination of the circle circumscribed to a polygon (triangle, square and pentagon); he shows how to divide a segment in equal parts; he teaches the geometric construction of the square root; he solves elementary problems of quadrature; he shows how to draw a circle whose area equals that of a square whose side is given; he teaches how to divide a circle in parts and conversely how to find a circle that is ½, 1/ 3, 2/3, etc, of a given circle; he also makes reference to the problem of the duplication of the cube and the determination of other multiples of it.

Only one chapter is devoted to planimetry[14]. In it, Pérez de Moya teaches the determination of areas of polygons (from the triangle to the polygon of 15 sides), of

[11] See "El Bachiller Ivan Perez de Moya a los lectores", letter written in 1567, December [Pérez de Moya 1568].

[12] They are about 760 pages: 271 in the first book and 487 in the second.

[13] See "El Bachiller Ivan Perez de Moya a los lectores" [Pérez de Moya 1568].

circular and oval figures, and applies this knowledge to the solution of such practical problems as the measuring of fields and farms, of cloth tissue, of the material needed to pave a floor or to construct a roof, and so on.

In 1573, Pérez de Moya published in Alcalá de Henares the *Tratado de Mathematicas: en que se contienen cosas de Arithmetica, Geometria, Cosmographia y Philisophia* [sic] *natural*, dedicated to the king of the Spanish Monarchy, Philip II[15]. It is a magisterial treatise that includes all the mathematics the author had previously published in Castilian, but also the main parts of his *Silva*, a work he published in Latin in 1557[16]. In his *Tratado de Mathematicas*, Pérez de Moya corrected some errors made in his previous works and offered additional information on some subjects[17]. It is divided in three parts. The first is dedicated to arithmetic and algebra. The second is a treatise on geometry, *Tratado de Geometria practica y speculativa*. The last one, *Tratado de cosas de Astronomia, y Cosmographia, y Philosofia natural*, includes different topics of astronomy, cosmography, and natural philosophy. Because of the variety and quantity of the information offered in the treatise, it appears that the author had eventually managed to find the resources necessary to publish the encyclopaedic work he could not published years before.

In his letter to the reader, Pérez de Moya stresses the importance of geometry in "all kinds of letters" ("todo genero de letras") and in the "mechanical arts" ("artes mechanicas"), adding that it is necessary for almost any job. He thinks that geometry is important to make the "Logician" understand some propositions studied at schools; it is

[14] This chapter is divided into "artículos".

[15] [Pérez de Moya 1573].

[16] In Moya's words: "Y asi va agora este libro como una summa de lo que se há hecho en lengua vulgar, y lo mejor y mas importante de las cosas naturales que pusimos en un librillo de Latin intitulado *Sylva*" [Pérez de Moya 1573], in "El Bachiller Ivan Perez de Moya, al letor". The Latin work Moya mentions here bears the title *Silva, eutrapelias id est Comitatis et urbanitatis ex variis probatae fidei Authoribus & vitae experimentis* [Seville 1557].

[17] Ob. Cit. in "El Bachiller Ivan Perez de Moya, al letor".

useful to the "Legist" because it teaches how to divide champs and farms; it is necessary for the "Soldier" and the "Astrologer" to calculate distances and measure altitudes and depths; finally to the "Architects" and all the other "jobs" (*officios*), which will find many important topics in it[18].

The *Tratado de Geometria practica, y speculativa* has a little more than 250 sheets. The first book concerns elementary geometry (see below); the second deals with altimetry, called "the first kind of measure"; the third "deals with things belonging to the second kind of measure, called planimetry"; and the last with stereometry.

The geometrical contents of the 1568 *Fragmentos Mathematicos* are not easily compared with those of the 1573 *Tratado de Geometria practica, y speculativa*. From one work to the other the author has rearranged the chapters and, sometimes, reorganized the contents of corresponding chapters. In any case, the *Tratado de Geometria* includes practically everything that appears in the *Fragmentos*, while some topics studied in the *Tratado de Geometria* are missing from the *Fragmentos*. This is the case with some questions dealing with the construction of polygons; with the determination of the difference between the polygons with the same number of sides that are circumscribed and inscribed to a circle; with the "proportion" between the circle inscribed into a geometrical "figure with many sides" and the corresponding circumscribed circle. In planimetry, the 1573 *Tratado* includes considerations about the areas of geometric solids not found in the 1568 *Fragmentos*.

Pérez de Moya's main concern is always practical and didactical, which leads him to put convenience and expedience on top of everything else. He warns the reader that sometimes his language is not above criticism as concerns strict mathematical truth,

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[18] Moya states several propositions as, for example: "Every triangle has three angles which sum is equal to two right angles" ("Omnis triangulus habet três ângulos equales duobis rectis") [Pérez de Moya 1584, p. 44-45).

and yet he is intentionally using it because it seems most suitable for the tyro. This is in particular the case when he deals with the side and the diagonal of the square, or when he explains how to calculate the area of the circle. As he knows his rivals may criticize him for "lack of rigour", he acknowledges his text contains some "improprieties":

> (…) auisoles que podran luego examinar algunas impropriedades que aquí hallaran, assi como decir en el capitulo diez y nueve del primero li[bro]. d[e] Geometria, que muestro saber el diametro de un cuadrado por la costa [i.e. the side], en lo qual como sea lo uno incommensurable, con lo otro, es impropriedad usar destre nombre saber. Y mas adelãte digo, q̄ muestro la quadratura del circulo: como no sea posible saber-se precissamente. Mas use destes terminos por decir dello a los principiãtes, lo que humanamente se puede hazer[19].

The *Principios de geometria* (1584)

The *Principios de geometria* is divided in two books (*Libros*), the first of which occupies the first 72 sheets and the second the remaining 55. We ignore the title of the first book, as the first sheet of text is missing. The second book "Deals with things concerning that kind of measure called Planimetry, which concerns the measuring and dividing of lands". Both books are organized in Chapters (*Capítulos*), and a few of them are still divided in Articles (*Artículos*)[20]. As Moya acknowledges, his *Principios* of 1584 contains a simplified version of the second book of his 1573 *Tratado de Geometria*.

Pérez de Moya's *Principios de geometria* is a very practical work specifically aiming to an audience of surveyors, explicitly mentioned by the author as the ones who

[19] See "El Bachiller Ivan Perez de Moya, al letor", in [Pérez de Moya 1573].

[20] The first *Libro* has 28 chapters, of which only *Capitulo* XXVII is further subdived in 10 articles. The *Libro segundo* has 23 *Capítulos*, and only the XVII is divided in 6 *Artículos*.

would most profit of its second book[21]. The small size of the volume, the reduced number of diagrams it contains, the poor quality of the printing generally (particularly as compared with the 1573 *Tratado*), the format and style of its presentation (see below), all leads us to suggest that the work must have been a cheap one, and one easily packed and moved around by its users. The price is missing in the place left to announce it[22]. In any case, in different places we find (and we do not find it in Moya's other geometrical works) an explicit preoccupation with providing detailed, specific, practical, and non-equivocal rules to solve specific mensuration problems, and also the explicit dismissal of problems that would have a mere theoretical interest. For instance, in dealing with the determination of the area of a triangle, Pérez de Moya gives only one method, because he wants the user not to be confused (in other works he had provided different methods):

> Para medir triângulos, ay tantos modos, y primores que quererlos referir aqui seria confundir los entendimientos de algunos medidores, con los muchos preceptos, los quales por averlos puesto en outro volumen, solo pondre una regla general para medir cualquier triangulo de qualquiera suerte y genero que sea, con solo la noticia de sus lados[23]

In another place, Pérez de Moya mentions the calculation of the areas of parts of a circle, which calculation he has provided in another work. Here, he does not want to deal with the subject since it is not of interest to surveyors: "[this calculation] no pongo aqui, porque no haze al propósito al medidor de tierras, para quien mi intento principal fue escrevir este segundo libro."[24].

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[21] [Pérez de Moya 1584, Segundo Libro, Cap. XIII].

[22] In the front page we can see "Tassado a … el pliego", although the price is not mentioned.

[23] [Pérez de Moya 1584, Segundo Libro, p. 85]

[24] [Pérez de Moya 1584, Segundo Libro, Capítulo XIII, p. 89v].

Although in the beginning of the *Principios de geometria*, Pérez de Moya points out that the work is a summary of the second book of his 1573 *Tratado de Geometria*, it is not the case that Moya just took some chapters from his great treatise and assembled them them together to "write" his new book. In fact, although it is true that the contents of *Principios de geometria* are found in the *Tratado de Geometría*, there are enough differences between them to make the *Principios* a new or different work. We notice among the differences the more detailed explanation of some subjects in the *Principios de geometria*. For instance, it devotes three chapters to carefully develop the properties of the triangles (IX, XII and XIII), while the *Tratado de Geometria* quickly summarizes them in only one chapter (XVI). We notice also Moya's concern with the applications of geometry to practical matters. Thus Moya teaches how to make an *esquadra*, a material instrument based on the concept of perpendicularity[25]. He takes care that no unknown notion becomes an obstacle for anyone lacking the basics. He thinks, for instance, that in the *Principios de geometria* he must introduce angles before dealing with the division of a segment in two or more parts—while in the *Tratado de Geometria* he had introduced them afterwards. Furthermore, the *Tratado de Geometria* as well as the *Fragmentos Mathematicos* includes a concluding section of solved problems (31 in the first and 28 in the second), where geometric concepts are applied to altimetry, planimetry and stereometry. However, in the *Principios de geometria* the examples follow immediately the study of each the corresponding subject.

Sources and some results

Pérez de Moya quotes a rather large number of sources along the text, sometimes by mentioning only the author's name, without reference to his work. With difference,

[25] [Pérez de Moya 1584, p. 46-47]

Euclid is the author most often quoted. Sometimes, the *Elements* is not explicitly mentioned but the numbers of the Book and proposition to which Moya is making reference accompany his name. The first Book of the *Elements* is the one most cited in the two books of the *Principios de geometria*; but there are also references to Books III, IV and VI (in *Principios de geometria*'s first book) and to Books II and V (in *Principios de geometria*'s second book). Pérez de Moya does not specify which edition of the *Elements* he uses, but Campano's translation is the most likely one, since in the *Fragmentos Mathematicos*, Campano's *Elements* is explicitly mentioned in one comment addressed to propositions 2 and 11 of Book VI:

> De outra manera dividiras una línea en las partes que quisieres como se infiere de la segũnda y onzena proposiciõ del 6. de Euclides (segun Campano)[26].

We know that Moya was also familiar with Zamberti's edition of the *Elements*, since he mentions it in the *Arithmetica practica y speculatiua* [Baranda 1998, p. 83]. Besides Euclides, Pérez de Moya mentions other authors. Aristotle's *Physics*, Book II, is mentioned in relation to the sum of the angles of a triangle. Archimedes appears in connexion with the relation between the area and the circumference of a circle. Ptolemy's *Almagest* in mentioned in a geometrical demonstration (see below). Sebastión Serlio and one Pinola appear in references to architecture[27]. Vitruve (libro 3, cap. 1), Columela (libro 5, cap.1) and Pliny (livro II, cap. 23, doubtless of the *Natural History*) are mentioned in relation to units of measure[28]. We are sure that Pérez de Moya

[26] [Pérez de Moya 1568, p. 61].

[27] Sebastión Serlio was a Bolognese architect and painter (1475-1554). Pérez de Moya does not mention the title of Serlio's work, but we know that he was the author of a treatise in six parts, *Regale generali di architettura* (1537-1551).

[28] Pérez de Moya does not mention titles of these author's works, but in all probability he was making reference to Vitruve's *De architectura*, Columela's *Res Rustica*, and Pliny's *Naturalis Historiae*.

knew many more sources for in the geometrical part of the *Fragmentos Mathematicos* of 1568 he had already quoted Peletier, Tartaglia, Cardano, Dürer, and Sacrobosco.

5. Some comments on the text

The first book presents the elements belonging to geometry (point, line, surface, solid, angle, term, figure, circle or circular figure, diameter, semicircle, portion of circle, sector of circle, area) and its foundations (definitions, petitions and common sentences). Interestingly, Pérez de Moya distinguishes the mathematical or abstract definition of a concept from its practical meaning or interpretation. For instance, in his definition of point we read:

> Punto es una cosa que no tiene parte, quiere dezir. Que es una cosa tan pequeña, q̃ su largura, y anchura, y profundidad, no se puede ver ni diuidir: en mitad, ni tercio, ni en otra ninguna parte por pequeña que sea: porque el pũto no es quantidad, mas un termino simple imaginado intencionalmente: para denotar el principio, medio o fin de alguna linea. Y por esto dizen, que el punto es una cosa que no ocupa lugar ni se puede ver, ni dividir en partes, y desta s[u]erte lo entiende el Mathematico. El natural entiende por punto, una señal hecha con tinta o con otra cosa deste modo **.** El qual por muy pequeño, y delicado que se haga se puede ver, y dividir[29].

And, in his definition of line:

> Linea, que en Español dezimos raya generalemente hablãdo[,] es una largura sin anchura ni profundidad. Los terminos o fines dela qual son dos puntos. Dize que sus fines o estremos son dos puntos: para differencia dela linea circular, que carece de terminos. El origem dela linea se imagina dela estension que intẽncionalmente se finxe correr de

[29] [Pérez de Moya 1584, p. 2[v]].

> un punto a outro. Y es una cosa la linea tan pequeña segũ su anchura (que puesto que es raya imaginada quan larga quisieremos) que no ay cosa por delicada que sea, que no tenga mayor groseza y anchura. Entiendesse la linea como el punto en dos modos uno según el Mathematico, y otro según el natural. El Mathematico entiende por linea una cosa que teniendo largura no tiene anchura ni profundidad, ni en ellas estas dos cosas se pueden ver, porque solo se magina su anchura intẽncionalmente como el entendim[i]ento para denotar cõ ella el principio, o m[e]di[o], o fin de una superficie. El natural entiende por linea una raya hecha con tinta, o sin ella desta manera. ——— La qual por delicada y sutil que se haga se puede su anchura ver y dividir [Pérez de Moya 1584, p. 3[v]].

Euclid is surely the "mathematician" mentioned in the text; as we see, Pérez de Moya's first definitions of point and line follow those of the *Elements*, although we also find a reference to the line as being the "path" of a point in motion. His definition of right line makes it the shortest path between two points:

> Linea recta, q̃ quiere dezir derecha, es una brevissima ẽxtension de un punto a otro, que recibe a los dichos puntos en sus extremos. De suerte, que si de un punto a otro con el entendimiento, o con tinta o con otra cosa se echaren rayas, la que fuere por el mas breve camino que ser puede se dia recta, o derecha, y todas las que no fueren por este mas breve camino se diran curvas, o tuertas, o torcidas.[Pérez de Moya 1584, p. 3[v]].

Next, the *Principios de geometria* deals with elementary issues on angles, segments of straight line (how they can be divided in two or more parts, how we can subtract them, etc), and triangles (its construction and classification). For a given triangle, it is specified the fundamental relation between its sides, and between its angles. Next, it deals with the construction of quadrilaterals, the regular pentagon, and other regular polygons with more than five sides.

In dealing with square roots, Moya teaches how to determine them geometrically, as the mean proportional between two segments. His exemples are √12, which he determines as the mean proportional between 4 and 3, and √7, mean proportional between 1 and 7 (the mean proportional he finds through the semicircle whose diameter is the "addition" of the known proportional terms, 3+4 and 1+7). In his *Fragmentos Mathematicos*, Pérez de Moya remarked that while in general it is not possible to find the exact root of a number, roots could be obtained exactly geometrically, if numbers "were reduced to line quantities" (*reduziendo el tal numero a quantidades de línea*) [Pérez de Moya 1568, p. 63-64]. Moya mentioned here "book I of volume II of Nicolo Tartaglia", in a likely reference to Tartaglia's *General Trattato di numeri, et misure* (1556).

As already pointed out, Moya pays much attention to planimetry and the determination of areas, the transformation of a triangle in a circle with the same area, the division in equal parts of grounds of various shapes (triangular, quadrangular, hexagonal, circular); and the addition and subtraction of polygonal figures. As is typical in works of this kind, Moya also deals with the computation of distances (between two distant points, heights, and so on). We also find a rather comprehensive list of units of measure, with their equivalences: "onça, palmo, dicha, espithema, deunx, pie, paso, passada, pertica, orgya, codo, plethrum, iugero, estádio, diaulos, dolicos, schenus, parafanga, stathmos, milha romana, milla alemã, milla grade, lapis". See Appendix for more details. [Pérez de Moya 1584, p. 73-75].

The applied orientation of the work is most evident when Moya distinguishes between two ways of measuring a field, if it is not horizontal. Slopping fields are to be measured af if they were horizontal when they are meant to be used for growing purposes. However, if they are meant for building purposes (for housing, we would say nowadays), then only the "basis" of the hilly field is to be measured:

> Nota mas, que donde ay monte, si el monte fuere para sembrar, midase como las demas superfícies planas. Y si fuere para dar termino a algun pueblo midase según su basis (si la venta no mandare otra cosa) [Pérez de Moya 1584, p. 100 [v]].

In his drawing (see Figure 1), the slopping field seen from the side is *ab*, and the "basis" is *db*.

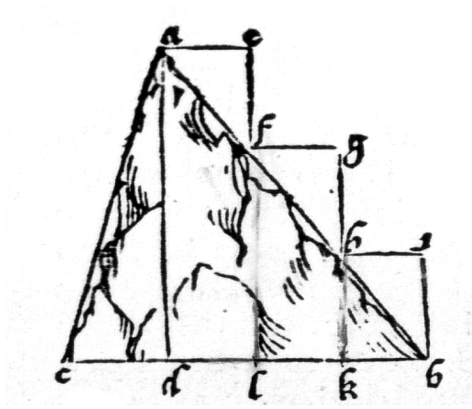

Figure 1[30]

To measure *db*, Moya suggests to horizontally hold out a stick of known length, *ae*, in the highest point of the field (which he assumes has a regular form). Then, from the end of the stick, *e*, a plumb line must be drop until it meets the sloping ground, say at *f*. Then we repeat the procedure until we reach the bottom line at *b*. The basis *db* will be equal to the addition of the lengths *ae*, *fg*, *hi*, etc. [Pérez de Moya 1584, p. 100 [v] – 101[v]].

Let us stress that in introducing the measure of the diagonal of the square, Moya provides a confused and ultimately wrong notion of incommensurability. In fact, he

[30] We reproduce here the figure Moya used in his *Tratado de Geometria* of 1573 (p. 144), identical to the one he used in the *Principios* of 1584.

makes the reader believe the diagonal is incommensurable with the side of the square because it can never be expressed as a whole number of sides:

> E notaras que jamas las diagonales de los quadrados vendran a ser comensurables com sus lados, quiero dezir, q la diagonal nunca tendra tamanos semejantes a los que el quadrado tuviere por lado justamente. No quiero dezir, que la diagonal hade ser tanto como el lado, q esto ninguno lo duda, porq cosa clara es ser la diagonal siempre mayor, o mas larga, sino q si por lado tuviere el cuadrado 10 tamanos o lo q fuere justamete, digo q la diagonal del cuadrado, no vedra a tener numero entero justamete de tamanos semejantes a los q tiene por lado, sino q vendra alguna fracion co los tamanos que tuviere. [Pérez de Moya 1584, p. 51-52].

Pérez de Moya studies in detail the circle and the circumference, pointing to their useful role in the construction of astrolabe sheets, in gnomonics, and many other things:

> Para la matéria de relojes, y para asentar en plano los vie[n]tos y hazer lanternas segun dizen los carpinteros, y para otras varias cosas se ofrece muchas vezes necesidad de dividir la circunferencia de un circulo en algunas partes yguales. [Pérez de Moya 1584, p. 63].

Pérez de Moya gives rule and compass constructions for the division the circumference in n equal parts, n = 2, 3, 4, 5, 6, 7, 8, 10, 12, 16, 24, 32 e 36, which only in some cases are exact, obviously. These constructions do not appear in the *Fragmentos Mathematicos*, and while they are included in the *Tratado de Geometria practica, y speculativa*, some are given differently. An interesting case in point is the division of the circumference in five equal parts, where Moya's construction is easy and simple but only yields a very roughly approximate result. He does not make any reference to Euclid's or Ptolemy's exact constructions[31]. Pérez de Moya's procedure to divide the circumference in five equal parts draws two perpendicular diameters (Figure 2); next, he divides one of the diameters (say, [*a b*]) in three equal parts, getting the points *e* and *f*; then, from one end of the second diameter, say *x*, we trace two cords through *e* and *f*, which cut the circumference at *g* and *h*. These points define an arc that Moya says is the fifth part of the circumference. [Pérez de Moya 1584, p.62[v]-68]

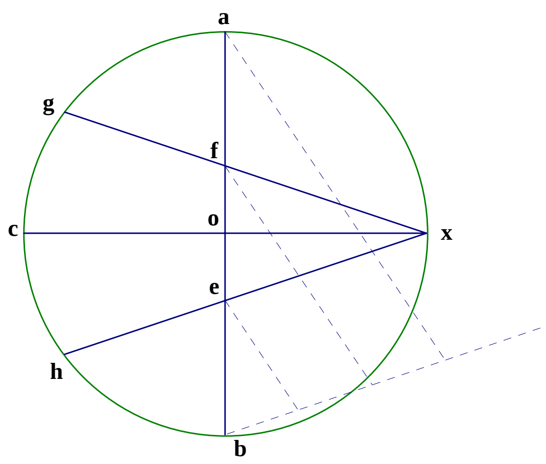

Figure 2

If the construction were correct, the segment *gh* would be equal to the side, *l*, of the pentagon inscribed in the circumference, and equal to the radius, *ox*. Let us call *d* the diagonal, *gx*. By construction, $of = \frac{1}{3}ox$, therefore, by similar triangles,

$\frac{\frac{1}{2}}{gx} = \frac{\frac{1}{3}ox}{\sqrt{\left(\frac{1}{3}ox\right)^2 + ox^2}}$. Hence, $\frac{\frac{1}{2}}{d} = \frac{\frac{1}{3}}{\frac{\sqrt{10}}{3}}$, and therefore $\frac{d}{l} = \frac{\sqrt{10}}{2}$, which is absurd, because, as we know, $\frac{d}{l} = \frac{1+\sqrt{5}}{2}$.

[31] Euclid, *Elements* IV, 11; Ptolemy, *Ptolemy's Almagest*, G.J. Toomer, ed. and trans. (London: Duckworth, 1984), I.10, p. 48-9.

Pérez de Moya's procedure to divide the circumference in three, seven and eleven parts draws, first, the radius, *eH* (see Figure 3); next, with centre *e* and radius *eH* Moya draws the *arc aHd*. Obviously, arc *aed* is a third of the circumference. Then, according to Pérez de Moya, half of the cord *ad* provides the side of the regular heptagon, and one third of the cord *ad* provides the side of the 11-side regular polygon. Pérez de Moya adds that it is possible to prove with the help of the compass that these constructions are in practice nearly correct.[Pérez de Moya 1584, pp. 62[v]-68].

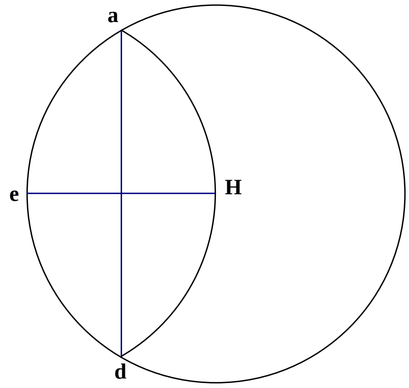

Figure 3

To conclude, let us turn to Pérez de Moya's determination of the area of the circle in the *Princípios de geometria*. Pérez de Moya points out that to measure the area of a circle is to find out how many "little squares" (*quadradicos*) of side equal to one foot it is possible to mark or make up within the circle. To know that number, it is enough to know the diameter or the perimeter of the circumference: "para medir la área de un circulo, se há de tener noticia de su diâmetro o de su circunferência porque con qualquiera cosa destas dos se media."[Pérez de Moya 1584, pp. 62[v]-68]. Moya quickly shows in his examples that this "number" of *little squares* (*quadradicos*) is not necessarily an integer number. In explaining the computation of the area of a circle whose perimeter is equal to 22 feet (he calculates it by taking the area equal to 7/88 times 22^2), Pérez de Moya only gives a practical rule that relates the area of a circle with the perimeter of its circumference:

> The reason of this operation is, that Archimedes proves that all the area of a circle is the seven, eighty eight fractions of the square of its circumference[32].

Later on, in the middle of an example, he mentions Archimedes's theorem (from *On the Measure of the circle*) that every circle is equivalent to a triangle rectangle whose base is equal to the perimeter of the circle and whose height is equal to the radius. Although the rule above can obviously be deduced from it, this is not pointed out. However, the relation between this theorem of Archimedes and the quadrature of the circle is explicit in Moya's treatises of geometry of 1568 and 1573.

Acknowledgments

We would like to thank António Machiavelo for his comments on a first draft of this paper. We also thank the library of the Instituto de Estudios Giennenses for providing us with copies of [Valladares Reguero 1997], and by [Leal y Leal 1972]. We gratefully acknowledge the support of Fundação para a Ciência e Tecnologia (FCT) through the Center of Mathematics of the University of Porto (CMUP) and the programme POCI/MAT/61237/2004 (to M.C.S.), and of the Spanish MEC (Hum2005-05107/FISO) and the Catalan DURSI (2005SGR-00929) (to A.M.).

[32] "La razon desta operación es, que Archimides demuestra que toda a area de un circulo es los siete, ochenta y ocho abos del cuadrado de su circunferencia" [Pérez de Moya 1584, p. 90].

APPENDIX: PÉREZ DE MOYA'S UNITS OF MEASURE

According to Moya his measures originated among the Romans and are grounded upon the basic Roman unit, the "finger" (*dedo*): "El origen de donde salen las diversas medidas, de que los Geometras, y Cosmographos se sirven, y de las que usaron los Romanos, es una medida q̃ diz ẽ dedo, y por dedo ent ẽ dian espacio desta linea M——L" [Pérez de Moya 1584, p. 73[v]]. We notice that the line has an approximate length of 2 cm. In his *Fragmentos Mathematicos*, Moya describe the *dedo* as the space occupied by 4 oat seeds: "espacio que occupan quatro granos de cevada". He also adds a figure whose length is very similar to the one we find here and is accompanied by a similar explanation. The following equivalences appear in [Pérez de Moya 1584, p. 73[v]-75].

Onza – 3 *dedos* (in his 1568 book, he has *onça* = 12 oat seeds)
Palmo – 4 *dedos* (Pérez de Moya, following Vitruve, stresses that his "palmo" is not equal to the length of the hand with extended fingers, but just the length of the flat hand surface: not the "mano estendida" but just "la palma de la mano")
Dicha – 2 *palmos*
Deunx – 10 *dedos* (Moya says this comes from Columela, *Res Rustica*, book 5, ch. 1)
Espithema – 3 *palmos*
Pie – 4 *palmos*, or 16 *dedos*
Paso – 2 *pies*
Pasada – there are two kinds of it: the *pasada comun* is the same as the *paso*. The *pasada geométrica* is equal to 2 *pasos*
Pertica – 10 *pies*
Orgia – 6 *pies*
Plethrum – 100 *pies*
Iugero – 100 *pies* ("*Iugero*" and "*Plethrum*" are two different names for the same thing which Moya mentions without further ado; he had mentioned them already in his *Tratado de Geometria* of 1573, p. 97)
Estadio – 125 *pasos geométricos* (Moya says this comes from Pliny)
Diaulos – 2 *estadios*
Dolicos – 12 *estadios*
Parafanga – 30 *estadios*

Schenus – 60 *estadios*

Milla or *Milla Romana* – 8 *estadios* = 1000 *pasos*

Milla Alemana comun (*German common mile*) – 32 *estadios* = 4000 *pasos*

Milla grande (*Long mile*) – 5000 *pasos*

Lápis – equal to the *milla romana*

Estadal — a square of side equal to $3+2/3$ of a "vara" (Pérez de Moya does not give here the length of the "vara", but in his *Tratado de Geometria, practica, y especulativa* (1573, p. 97), 1 *vara* = 3 *pies*)

Ulna comun – 4 *pies*

Ulna agreste – 6 *pies*

Cubito, ou *codo pequenõ* - 1 and a half *pie*

Codo comum- 8 *palmos*

Codo grande- 16 *palmos*

Legua – one and a half *milla* = 12 *estadios*

Legua comum – 3 *millas* = 24 *estadios*

Legua Española – 4 millas = 32 *estadios* = 4000 *passos*

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