Time-dependent problems for a wedge via Kontorovich-Lebedev's transform

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Abstract

The aim of this paper is to solve a certain type of time-dependent problems for a wedge in terms of the Kontorovich-Lebedev integral. In order to do that we will consider a certain testing-function space for distributions associated with the two-dimensional Kontorovich-Lebedev transformation.

Keywords: Testing-function spaces, distributions, multidimensional Kontorovich-Lebedev transform, modified Bessel functions, time-dependent problems.

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1 Introduction

Time-dependent problems are a challenge for the comprehension of many phenomena of mathematical-physics. The solution of this type of problems is of great importance for a large variety of practical applications. They can be very useful in evaluating effects of the exposition of a complex structure to impulsive electromagnetic excitations, as for example, in the cases of the lightning flash or the nuclear pulse [5], or in the analysis of acoustic waves [10]. In fact, it is expected that a solution to the time-dependent problems contain many of the elementary patterns from the stationary solution. The main difference is that the patterns will be slowly varying in time instead of time-independent and, consequently, the elementary wave patterns may bifurcate. An example of this occurs when a shock is incident on a wedge [5] and it is connected, for example, to the study of first passage behavior of fractional Brownian motion in two-dimensional wedge domains [4] or to the scattering of an electromagnetic time-dependent plane wave by the edge of an impedance wedge [6].