A class of index transforms generated by the Mellin and Laplace operators

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Abstract

Classical integral representation of the Mellin type kernel

$$x^{-z} = \frac{1}{\Gamma(z)} \int_0^\infty e^{-xt} t^{z-1} dt, \ x > 0, \ \text{Re z} > 0,$$

in terms of the Laplace integral gives an idea to construct a class of non-convolution (index) transforms with the kernel

$$k_z^{\pm}(x) = \int_0^\infty \frac{e^{-xt^{\pm 1}}}{r(t)} t^{z-1} dt, \ x > 0,$$

where $r(t) \neq 0, \ t \in \mathbb{R}_+$ admits a power series expansion, which has an infinite radius of convergence and the integral converges absolutely in a half-plane of the complex plane z. Particular examples give the Kontorovich-Lebedev-like transformation and new transformations with hypergeometric functions as kernels. Mapping properties and inversion formulas are obtained. Finally we prove a new inversion theorem for the modified Kontorovich-Lebedev transform.

Keywords: *Mellin transform, Laplace transform, Kontorovich-Lebedev transform, modified Bessel functions, hypergeometric functions*

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