

Dynamical Systems Seminar

Date. October 26, 14h30

Place. Room M031

Speaker. Sandra Vaz (Universidade da Beira Interior)

Title. Discrete Dynamical Systems and Number Theory

Abstract. Different research directions can arise by using number theory in the study of dynamical systems properties.

Firstly, we study, T_β , the family of β -transformations,

$$\begin{aligned} T_\beta : [0, 1[&\rightarrow [0, 1[\\ x &\rightarrow \beta x - \lfloor \beta x \rfloor; \quad \beta \in]1, 2] \end{aligned}$$

and the tent family, τ_s ,

$$\tau_s(x) = \begin{cases} sx & \text{se } 0 \leq x \leq 1/2 \\ s(1-x) & \text{se } 1/2 < x \leq 1 \end{cases}; \quad s \in]1, 2]$$

using kneading theory, a technique from symbolic dynamics. The nature of the parameters provides a different behaviour, so we consider different classes of algebraic integers to analyze it.

Another application of symbolic dynamics is the representation of real numbers in different bases. Given $\beta > 1$, an algebraic integer, the iteration of T_β gives rise to the well known greedy β -expansion. Its properties have been widely studied in the literature.

K. Schmidt analyzed $\text{Per}(\beta)$, the set of periodic points of T_β , when $\beta^2 = n\beta + 1$, $n \geq 1$. In an attempt to generalize his result, we present a new representation (we call *linear expansion*)

$$x = \sum_{i \geq 1} e_i \beta^{-i},$$

where e_i can be larger than $\lfloor \beta \rfloor$, and its relationship with $\text{Per}(\beta)$.

Remark. Coffee with the speaker is served after the talk (15h30 - 16h00)