Seminário de Geometria e Topologia

Singularities of the vector fields generated by Implicit differential equations

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7 de Abril, sexta-feira, 15h00, sala 0.04

Consider the system of implicit differential equations:

$$F(t, x, p) = 0, \quad p = \frac{dx}{dt}, \tag{1}$$

where $t \in \mathbb{R}$ and $x = (x_1, \ldots, x_n)$, $p = (p_1, \ldots, p_n)$, vector-function $F = (F_1, \ldots, F_n)$ is smooth and has maximal rang n. A point from $(2n+1)$-dimensional space (t, x, p) is called singular point of (1), if the matrix $\partial F/\partial p$ in this point is degenerate, i.e., its determinant vanishes. In the neighborhood of the singular point system (1) could not be reduced to the explicit system, and it defines some multivalent direction field in space (t, x). We will use the lifting of this multivalent direction field on the manifold $\mathfrak{F} : F(t, x, p) = 0$ in space (t, x, p). After the lifting we obtain one-valued direction field $\bar{l}$ on $\mathfrak{F}$ (this construction is similar to Riemann surface for the multivalent function of complex variable). Projection $\pi : \mathfrak{F} \to (t, x)$ turns the trajectories of lifted field $\bar{l}$ to integral curves of the system (1). This construction firstly was used by Poincaré in the case $n = 1$.

In this talk we will consider the system (1) with $n > 1$. In this case lifted direction field $\bar{l}$ on the $(n+1)$-dimensional manifold $\mathfrak{F}$ is given by vector field v with special properties. To be exact, the components of the vector field v are connected with functional relations. In the general case on the manifold $\mathfrak{F}$ there exist local coordinates $(\xi, \eta, z_1, \ldots, z_{n-1})$, such that vector field v is given by formula:

$$\dot{\xi} = v, \quad \dot{\eta} = w, \quad \dot{z}_i = \alpha_i v + \beta_i w \quad (i = 1, \ldots, n-1), \tag{2}$$

where v, w and α_i, β_i are appropriate smooth functions of variables $(\xi, \eta, z_1, \ldots, z_{n-1})$. Singular points of the field (2) are not isolated, they form $(n-1)$-dimensional submanifold $W^c \subset \mathfrak{F}$ given by equations: $v = 0$, $w = 0$. In each singular point linearization of the field (2) has eigenvalues $(\lambda_1, \lambda_2, 0, \ldots, 0)$. If $\operatorname{Re} \lambda_{1,2} \neq 0$, then W^c is central manifold of the vector fields (2), and from Shoshitaishvili's reduction theorem follows, that (2) is topologically equivalent to the simple normal form: $\dot{\xi} = \pm\xi$, $\dot{\eta} = \pm\eta$, $\dot{z}_i = 0$ $(i = 1, \ldots, n-1)$. In this talk I am planning to present some theorems about finite smooth normal forms of the vector field (2) and describe phase portraits of this field in the neighborhood of it singular points.