Invariance principles, large deviations, and convergence of fast-slow ODEs to stochastic differential equations

Abstract:

A project started recently with Andrew Stuart (Warwick) investigates the convergence of certain deterministic systems to a stochastic differential equation. As a starting point, we consider the (over-simplistic) fast-slow system

$$\dot{x} = \epsilon^{-1} f_0(y) + f(x, y)\dot{y} = \epsilon^{-2} g(y)$$

where the fast y-variables lie in an attractor with invariant measure μ and the slow x-variables lie in $\mathbb{R}^d$. It is assumed that $\int f_0 \, d\mu = 0$.

We prove that under very mild conditions on the fast variables, solutions x^ϵ converge weakly in $C([0, T], \mathbb{R}^d)$ as $\epsilon \to 0$ to solutions X of the stochastic differential equation

$$\dot{X} = \dot{W} + F(X),$$

where W is a d-dimensional Brownian motion and

$$F(x) = \int f(x, y) \, d\mu(y).$$

A major difference between our approach and related projects is that we do not rely on decay of correlations for the $\dot{y}$ equation (decay of correlations for flows is a notoriously difficult and poorly understood problem). Instead we use invariance principles (a generalisation of the central limit theorem giving convergence to Brownian motion) and large deviation estimates which have been derived for a very large class of systems in collaboration with Matthew Nicol (Houston).