ON A PSEUDOSPECTRA OF TOEPLITZ MATRICES

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ABSTRACT. The spectral points of a nonhermitian Toeplitz matrix A, with the dimension N are usually highly sensitive to perturbations. An equivalent statement is that the resolvent $(zI - A)^{-1}$ of a Toeplitz matrix may be much larger in norm, even when z is far from the spectrum. Because of these facts, the eigenvalues of nonhermitian Toeplitz matrices for any but the most theoretical purposes should be considered suspect. In many applications it is more meaningful to investigate the $\varepsilon-pseudoeigenvalues$: the complex numbers z with $\left\|(zI - A)^{-1}\right\| \geq \varepsilon^{-1}$. In this study, analyzes the pseudospectra $\sigma_\varepsilon(A)$ of Toeplitz matrices, and in particular relates them to the symbols of the matrices and thereby to the spectra of the associated Toeplitz operators. We write programs on Mathematica to plothing $\sigma(A)$ and $\sigma_\varepsilon(A)$ of Toeplitz matrices. Computed examples of pseudospectra are presented throughout, with using Mathematica and Matlab (with Eigen-tool), and applications in numerical analysis are mentioned.

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