Smoothing the Gibbs phenomenon using Hermite-Padé approximants

B. Beckermann[1], Ana Matos[2], F. Wielonsky[3]

[1]Université de Lille 1
email: Bernhard.Beckermann@math.univ-lille1.fr
[2]Université de Lille 1
email: Ana.Matos@math.univ-lille1.fr
[3]Université de Lille 1
email: Franck.Wielonsky@math.univ-lille1.fr

Abstract

In order to reduce the Gibbs phenomenon exhibited by the partial Fourier sums of a periodic function f, defined on $[-\pi, \pi]$, discontinuous at 0, Driscoll and Fornberg suggested the construction of a class of approximants which incorporate the knowledge of that singularity. More precisely, their approach is the following one: let g_2 denote the series such that $f(t) = \Re(g_2(e^{it}))$. Then, the goal is to approach g_2 on the unit circle (and more precisely its real part). It is typical that the singularity of the function f, located at 0 say, corresponds to a logarithmic singularity for g_2, then located at 1, and that this function g_2 is analytic in the complex plane, with a branch cut that can be taken as the interval $[1, \infty)$. Defining $g_1(z) = \log(1-z)$, we may consider the problem of determining polynomials p_0, p_1, p_2 such that

$$p_0(z) + p_1(z)g_1(z) + p_2(z)g_2(z) = \mathcal{O}(z^{n_0+n_1+n_2+2}) \quad (z \to 0)$$

where n_j denotes the degree of p_j, $j = 0, 1, 2$. We can then propose the *Hermite-Padé approximant*

$$\Pi_{\vec{n}}(z) = -\frac{p_0(z) + p_1(z)g_1(z)}{p_2(z)}, \tag{1}$$

to approximate g_2. Note that when $p_1(z) = 0$ (or formally $n_1 = -1$) we recover the usual Padé approximant of g_2 of type (n_0, n_2).

Convincing numerical experiments have been obtained by Driscoll and Fornberg, but no error estimates have been proven so far. In this talk we obtain rates of convergence of sequences of Hermite–Padé approximants for a class of functions known as *Nikishin systems*. Our theoretical findings and numerical experiments confirm that particular sequences of Hermite-Padé approximants (diagonal and row sequences, as well as linear HP approximants) are more efficient than the more elementary Padé approximants, particularly around the discontinuity of the goal function f.