Program and Abstracts

Conference
Non-Associative Algebras
and
Related Topics II

18-22 July 2022
Coimbra-Portugal

Foreword

We would like to welcome you to the NAART II (Non-Associative Algebras and Related Topics II) and wish you a pleasant stay in our town.

This conference is dedicated to honor Alberto Elduque on the occasion of his 60th birthday. In the past 30 years, Professor Elduque has significantly contributed to the progress of Algebra, notably to the theory of non-associative graded algebras. Our aim is to bring together those mathematicians who have been important in Elduque's scientific career, whether his professors, collaborators, students, or simply his fellow travellers in the journey of math. The venue will be the historic University of Coimbra, founded in 1290—the first university in Portugal and one of the oldest in Europe. The host is the Department of Mathematics of the University of Coimbra (DMUC), drawing on the support of several other Portuguese research institutions.

We are deeply indebted to the Center for Mathematics of the University of Coimbra, to the Center for Mathematics of the University of Porto, to the Center for Mathematics and Applications of the University of Beira Interior and to the Foundation for Science and Technology.

Helena Albuquerque
Chair of the Organizing Committee

Contents

1 List of Participants

Helena Albuquerque (University of Coimbra, Portugal)
María Alejandra Álvarez (University of Antofagasta, Chile) [Abstract] [Schedule]
José Angel Anquela (University of Oviedo, Spain) [Abstract] [Schedule]
Diego Aranda Orna (University of La Rioja, Spain) [Abstract] [Schedule]
Sebastiano Argenti (University of Basilicata, Italy) [Abstract] [Schedule]
Shavkat Ayupov (V.I. Romanovskiy Institute of Mathematics, Uzbekistan) [Abstract] [Schedule]
Per Bäck (Mälardalen University, Sweden) [Abstract] [Schedule]
Lucia Bagnoli (University of Bologna, Italy)
Yuri Bahturin (Memorial University of Newfoundland, Canada) [Abstract] [Schedule]
Sigiswald Barbier (Ghent University, Belgium) [Abstract] [Schedule]
Elisabete Barreiro (University of Coimbra, Portugal)
Patrícia Beites (University of Beira Interior, Portugal)
Saïd Benayadi (University of Lorraine, France) [Abstract] [Schedule]
Pilar Benito (University of La Rioja, Spain) [Abstract] [Schedule] Mini course:[Abstract] [Schedule]
Víctor Blasco Jimenez (Dokuz Eylül University, Turkey)
Sofiane Bouarroudj (New York University Abu Dhabi, UAE)
Jose Brox (University of Coimbra, Portugal)
Yolanda Cabrera Casado (University of Málaga, Spain)
Antonio Jesús Calderón Martín (University of Cádiz, Spain) [Abstract] [Schedule]
Luisa María Camacho (University of Sevilla, Spain) [Abstract] [Schedule]
Paula Carvalho (University of Porto, Portugal)
Alejandra Sarina Córdova Martínez (University of Zaragoza, Spain) [Abstract] [Schedule]
Teresa Cortés (University of Oviedo, Spain)
Isabel Cunha (University of Beira Interior, Portugal)
Alberto Daza García (University of Zaragoza, Spain) [Abstract] [Schedule]
Tom De Medts (Ghent University, Belgium)
Thiago Castilho de Mello (Federal University of São Paulo, Brazil) [Abstract] [Schedule]
Anneleen De Schepper (Ghent University, Belgium)
Jari Desmet (Ghent University, Belgium) [Abstract] [Schedule]
Stephen Donkin (University of York, UK)
Vladimir Dotsenko (University of Strasbourg, France)
Cristina Draper Fontanals (University of Málaga, Spain) [Abstract][Schedule]
Askar Dzhumadil'daev (Kazakh-British Technical University, Kazakhstan) [Abstract][Schedule]

Alberto Elduque (University of Zaragoza, Spain) [Abstract] [Schedule]
Eva Elduque Laburta (Autonomous University of Madrid, Spain) [Abstract] [Schedule]
Farkhod Eshmatov (V.I. Romanovskiy Institute of Mathematics, Uzbekistan)
Juan Carlos Fernandez (University of São Paulo, Brazil) [Abstract] [Schedule]
Ignacio Fernández Rúa (University of Oviedo, Spain) [Abstract] [Schedule]
Amir Fernández Ouaridi (University of Cádiz, Spain)
Vyacheslav Futorny (University of São Paulo, Brazil) [Abstract] [Schedule]
Mario Galici (University of Palermo, Italy)
Esther García (University Rey Juan Carlos, Spain) [Abstract] [Schedule]
Germán García Butenegro (Mälardalen University, Sweden) [Abstract] [Schedule]
Xabier García Martínez (University of Vigo, Spain) [Abstract] [Schedule]
Faber Alberto Gómez González (University of Antioquia, Colombia) [Abstract] [Schedule]
Miguel Angel Gómez Lozano (University of Málaga, Spain) [Abstract] [Schedule]
Santos González (University of Oviedo, Spain)
María Isabel Hernández (CONACYT-CIMAT Mérida, Mexico) [Abstract] [Schedule]
Paulien Jansen (Ghent University, Belgium) [Abstract] [Schedule]
Victor Kac (Massachusetts Institute of Technology, USA) [Abstract] [Schedule]
Arun Kannan (Massachusetts Institute of Technology, USA) [Abstract] [Schedule]
Iryna Kashuba (University of São Paulo, Brazil) [Abstract] [Schedule]
Mikhail Kotchetov (Memorial University of Newfoundland, Canada) [Abstract] [Schedule]
María Pilar Laburta Santamaría (University of Zaragoza, Spain)
Manuel Ladra (University of Santiago de Compostela, Spain) [Abstract] [Schedule]
Jesús Laliena (University of La Rioja, Spain)
Christian Lomp (University of Porto, Portugal)
Samuel Lopes (University of Porto, Portugal)
Sergio David Cía Luvecce (University of Sorbonne, France)
Manuel Mancini (University of Palermo, Italy) [Abstract] [Schedule]
Consuelo Martínez (University of Oviedo, Spain) [Abstract] [Schedule]
Fabrizio Martino (University of Palermo, Italy) [Abstract] [Schedule]
Fernando Montaner (University of Zaragoza, Spain) [Abstract] [Schedule]
Rosa María Navarro (University of Extremadura, Spain) [Abstract] [Schedule]

Natália Pacheco Rego (Polytechnic Institute of Cávado and Ave, Portugal) [Abstract] [Schedule]
María Pilar Páez Guillán (University of Wien, Austria) [Abstract] [Schedule]
Francisco José Palomo Ruiz (University of Málaga, Spain) [Abstract] [Schedule]
Victor Petrogradsky (University of Brasilia, Brazil) [Abstract] [Schedule]
Alejandro Piñera Nicolás (University of Oviedo, Spain)
Dora Puljic (University of Edinburgh, UK)
Jhon Alexander Ramirez Bermudez (University of Antioquia, Colombia) [Abstract] [Schedule]
Carla Rizzo (University of Coimbra, Portugal)
Jorge Roldán López (University of La Rioja, Spain) [Abstract] [Schedule]
Iván Ruiz Campos (University of Málaga, Spain) [Abstract] [Schedule]
Ana Paula Santana (University of Coimbra, Portugal) [Abstract] [Schedule]
Paulo Saraiva (University of Coimbra, Portugal)
Umut Sayin (Düzce University, Turkey)
Yoav Segev (Ben-Gurion University, Israel) [Abstract] [Schedule]
Ivan Shestakov (University of São Paulo, Brazil) [Abstract] [Schedule]
Sergei Silvestrov (Mälardalen University, Sweden) [Abstract] [Schedule]
Michiel Smet (Ghent University, Belgium) [Abstract] [Schedule]
Adriana Suárez Corona (University of León, Spain)
Rustam Turdibaev (V.I. Romanovskiy Institute of Mathematics, Uzbekistan) [Abstract] [Schedule]
Guillermo Vera de Salas (University Rey Juan Carlos, Spain) [Abstract] [Schedule]
Efim Zelmanov (Southern University of Science and Technology, China) [Abstract] [Schedule]

2 Schedule

SCHEDULE

Monday, July 18th

8:30 - 9:30:	Registration.
9:30 - 10:00:	Opening ceremony.
10:00 - 10:30:	Plenary session: Santos González, *Around Alberto Elduque.*
10:30 - 11:30:	Plenary session: Consuelo Martínez, *On finitely presented (super)algebras.*
11:30 - 12:00:	Coffee break.
12:00 - 13:00:	Plenary session: Vyacheslav Futorny, *Lie algebras of vector fields on algebraic varieties.*
13:00 - 15:00:	Lunch.
15:00 - 16:00:	Mini course 2: Victor Kac, *Algebraic structures arising in physics.* (Chair: Consuelo Martínez)
16:00 - 17:30:	Contributed talks.
17:30 - 18:00:	Coffee break.
18:00:	Visit to the University of Coimbra (Group 1).

Tuesday, July 19th

9:00 - 10:30:	Mini course 1: Pilar Benito, *Quadratic Lie algebras.* (Chair: Helena Albuquerque)
10:30 - 11:00:	Coffee break.
11:00 - 12:00:	Plenary session: Ivan Shestakov, *Non-matrix varieties and coordinatization theorems for non-associative algebras.*
12:00 - 13:00:	Contributed talks.
13:00 - 15:00:	Lunch.
15:00 - 16:30:	Mini course 2 (part two): Victor Kac, *Algebraic structures arising in physics.*
16:30 - 17:00:	Coffee break.
17:00 - 18:00:	Contributed talks.
18:00:	Visit to the University of Coimbra (Group 2).

Wednesday, July 20th

9:00 - 10:30:	Contributed talks.
10:30 - 11:00:	Coffee break.
11:00 - 12:00:	Plenary session: Yuri Bahturin, *Inner actions on simple algebras.*
12:00 - 17:00:	Conference lunch.
17:00 - 20:00:	Tour to Mata Nacional do Buçaco.

Thursday, July 21st

9:30 - 10:30:	Poster Session.
10:30 - 11:00:	Coffee break.
11:00 - 12:00:	Mini course 3: Fernando Montaner, *Lie bialgebras.* (Chair: Efim Zelmanov)
12:00 - 13:00:	Plenary session: Mikhail Kotchetov, *Gradings on simple algebras.*
13:00 - 15:00:	Lunch.
15:00 - 16:30:	Contributed talks.
16:30 - 17:00:	Coffee break.
18:00:	Visit to the University of Coimbra (Group 3).

Friday, July 22nd

9:00 - 10:30:	Mini course 3 (part two): Fernando Montaner, *Lie bialgebras.*
10:30 - 11:00:	Coffee break.
11:00 - 12:00:	Mini course 1 (part two): Cristina Draper, *In the footsteps of Alberto Elduque.* (Chair: Mikhail Kotchetov)
12:00 - 13:00:	Plenary session: Efim Zelmanov, *On growth of algebras.*
13:00 - 15:00:	Lunch.
15:00 - 15:30:	Plenary session: Helena Albuquerque and Jesús Laliena, *To be or not to be associative: that is the question.*
15:30 - 16:30:	Closing plenary session: Alberto Elduque, *Tensor categories, algebras, and superalgebras.*
16:30 - 17:00:	Coffee break.

All plenary sessions will take place in the Room Pedro Nunes.

3 Schedule of Contributed Talks

SCHEDULE OF CONTRIBUTED TALKS

Monday, July 18th

Room 2.4
(Chair: Alberto Elduque)
16:00 - 16:25: Esther García,
Gradings induced by nilpotent elements,
16:30 - 16:55: Eva Elduque,
Differential graded Lie algebras in Hodge theory,
17:00 - 17:25: Arun Kannan,
New constructions of exceptional Lie superalgebras using tensor categories.

Room 2.5
(Chair: Yuri Bahturin)
16:00 - 16:25: Ana Paula Santana,
Some functors preserving minimal projective resolutions,
16:30 - 16:55: Thiago Castilho de Mello,
Gradings, identities, and images of multilinear polynomials on matrix algebras,
17:00 - 17:25: Germán García Butenegro,
Hom-associative algebras and divisibility.

Tuesday, July 19th

Room 2.4

(Chair: Samuel Lopes)

12:00 - 12:25: Jorge Roldán López,
Lattices of ideals in quadratic Lie algebras,

12:30 - 12:55: Saïd Benayadi,
Quadratic structures on flat pseudo-Euclidean Lie algebras,

(Chair: Vyacheslav Futorny)

17:00 - 17:25: Sigiswald Barbier,
A deformed periplectic Brauer category,

17:30 - 17:55: Jari Desmet,
Non-associative Frobenius algebras for simple algebraic groups.

Room 2.5

(Chair: Fernando Montaner)

12:00 - 12:25: Per Bäck,
Non-associative skew Laurent polynomial rings and related rings,

12:30 - 12:55: Michiel Smet,
Generalized Kantor pairs.

(Chair: Teresa Cortés)

17:00 - 17:25: Paulien Jansen,
Abelian Tits sets and Jordan algebras,

17:30 - 17:55: Jhon Alexander Ramirez Bermudez,
Second cohomology group for finite-dimensional simple Jordan superalgebra $\mathcal{D}_t$ with $t \neq 0$.

Room 17 de Abril

(Chair: Patrícia Beites)

12:00 - 12:25: Victor Petrogradsky,
Phoenix restricted Lie algebras,

12:30 - 12:55: Sergei Silvestrov,
Color Quasi-Lie algebras and Hom-algebras.

(Chair: Saïd Benayadi)

17:00 - 17:25: Rosa María Navarro,
Complex cyclic Leibniz superalgebras,

17:30 - 17:55: Natália Pacheco Rego,
On some properties of generalized Lie-*derivations of Leibniz algebras.*

Wednesday, July 20th

Room 2.4
(Chair: Manuel Ladra)
9:00 - 9:25: Xabier García Martínez,
Two characterisations of Lie algebras,
9:30 - 9:55: María Pilar Páez Guillán,
On the subalgebra lattice of a restricted Lie algebra,
10:00 - 10:25: María Alejandra Álvarez,
On rigid 3-dimensional Hom-Lie algebras.

Room 2.5
(Chair: Ivan Shestakov)
9:00 - 9:25: Faber Alberto Gómez González,
Wedderburn principal theorem for Jordan superalgebras type $\mathfrak{Kan}(2)$,
9:30 - 9:55: Iryna Kashuba,
Free Jordan algebras,
10:00 - 10:25: José Angel Anquela,
Quadratic Jordan algebras generated by two idempotents.

Room 17 de Abril
(Chair: Paulo Saraiva)
9:30 - 9:55: Fabrizio Martino,
Differential identities and polynomial growth of the codimensions,
10:00 - 10:25: Pilar Benito,
Double extensions related to oscillator algebras.

Thursday, July 21st

Room 2.4
(Chair: Jesús Laliena)
15:00 - 15:25: Guillermo Vera de Salas,
Homogeneous superderivations with nilpotent values in semiprime associative superalgebras,
15:30 - 15:55: Yoav Segev,
A characterization of the quaternions using commutators,
16:00 - 16:25: María Isabel Hernández,
Low-dimensional commutative power-associative superalgebras,
16:30 - 16:55: Manuel Mancini,
Two-step nilpotent Leibniz algebras.

Room 17 de Abril
(Chair: Santos González)
15:00 - 15:25: Askar Dzhumadil'daev,
Weak Leibniz algebras,
15:30 - 15:55: Manuel Ladra,
Universal central extensions of compatible Leibniz algebras,
16:00 - 16:25: Rustam Turdibaev,
Poisson structure on the invariants of pairs of matrices,
16:30 - 16:55: Shavkat Ayupov,
2-Local and local derivations and automorphisms of Lie algebras.

4 Mini Courses

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Mini-Course on quadratic Lie algebras

Pilar Benito[1]

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A bilinear form φ on a nonassociative algebra A with product ab is said to be invariant iff $\varphi(ab, c) = \varphi(a, bc)$. Finite-dimensional complex semisimple Lie algebras with the Killing form and certain nonassociative algebras with the trace form carry such a structure. They are call pseudo-quadratic or pseudo-metrised algebras and, quadratic or metric, as long as φ is non-degenerate. The goal of the mini-course is to provide examples, basic structure and classical constructions on quadratic Lie algebras over the time.

Keywords: Lie algebra, quadratic, invariant form

References

[1] M. Bordemann, Nondegenerate invariant bilinear forms on nonassociative algebras, *Acta Math. Univ. Comenianae* **66** (1997), no. 2, 151–201.

[2] V. G. Kac, Infinite-dimensional Lie algebras, *Cambridge University Press,1990.*

[3] V. S. Keith, Groupes de Lie munies de métriques bi-invariants, *On invariant bilinear forms on finite-dimensional Lieie algebras, PhD Dissertation, Tulane University, 1984.*

[4] A. Medina, P. Revoy, Algebres de Lie et produit scalaire invariant, *Annales scientifiques de l'École Normale Supérieure* **18** (1985), no. 3, 553–561.

[5] G. Favre, L. J. Santharoubane, Symmetric, invariant, Non-degenerate bilinear form on a Lie algebra, *Journal of Algebra* **105** (1987), no. 2, 451–464.

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

In the footsteps of Alberto Elduque

Cristina Draper Fontanals[1]

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Pilar Benito and Alberto Elduque were my PhD supervisors. I chose Alberto because I had heard about his works on the 6-dimensional sphere as $G_2/SU(3)$ and the colors of quarks [1], actually more closely related to Malcev algebras than to the standard model!. All this exotic world attracted my attention so that a large part of the doctoral dissertation was devoted to G_2-homogeneous spaces, mainly from an algebraic viewpoint [2, 3]. Since then, I share with Alberto the fascination for exceptional algebras and groups, as well as for the big family of surprising objects which usually appear nearby. My purpose is to talk about how these ideas have marked my later work, often trying to understand better: which were those manifolds, why geometers did not seem to be interested in our work, whether that list of dimensions was pure chance... Time has not disappointed me and some answers have appeared in the way.

In any case, in his 60th birthday, I would like to thank him for his encouragement and the inspiration of his ideas...

Keywords: G_2, homogeneous spaces, exceptional Lie groups

Mathematics Subject Classification 2020: 17B25, 53C30

References

[1] Alberto Elduque and Hyo Chul Myung, Color Algebras and Affine Connections on S^6, *Journal of Algebra* **149** (1992), 234–261.

[2] Pilar Benito, Cristina Draper and Alberto Elduque, Models of the Octonions and G_2, *Linear Algebra Appl.* **371** (2003), 333–359.

[3] Pilar Benito, Cristina Draper and Alberto Elduque, Lie-Yamaguti algebras related to $\mathbf{g}_2$, *J. Pure Appl. Algebra* **202** (2005), 22–54.

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Algebraic structures arising in physics

Victor G. Kac[1]

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In the lectures we will discuss elements of the theory of vertex algebras, which are playing an increasingly important role in mathematics and in theoretical physics, and the related theory of Poisson vertex algebras and their multiplicative analogs, that are instrumental in the theory of integrable partial differential equations and differential difference equations.

Keywords: vertex algebras, Poisson vertex algebras

Mathematics Subject Classification 2020: 17B69, 17B63

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Lie bialgebras and related structures

Fernando Montaner[1]

[1]Departamento de Matemáticas, Universidad de Zaragoza,
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We will expose results on the notion and classification of Lie bialgebra structures, beginning from the basic notions up to the problem of their classification over current algebras, and we will mention recent advances and open questions on that problem.

Keywords: Quantum groups, Lie bialgebras, Drinfeld doubles

Mathematics Subject Classification 2020: 16Txx, 17B37

5 Plenary Sessions

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Inner actions on simple algebras

Yuri Bahturin[1]

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We would like to talk about some applications of the classification of group gradings on simple algebras to the study of actions of a class of Hopf algebras on simple algebras. Some of the results are from a joint paper with Susan Montgomery [1].

Keywords: Graded algebras, Hopf algebras

Mathematics Subject Classification 2020: 16T05, 16W50, 17B37

References

[1] Bahturin, Yuri; Montgomery, Susan, Group gradings and actions of pointed Hopf algebras, *J. Algebra Appl* **20** (2021), no. 01, 1-40.

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Tensor categories, algebras, and superalgebras

Alberto Elduque[1]

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After reviewing the basic definitions of tensor categories and the notion of semisimplification of symmetric tensor categories, it will be shown how the semisimplification of the category of representations of the cyclic group of order 3 over a field of characteristic 3 is naturally equivalent to the category of vector superspaces over this field. This allows to define a superalgebra starting with any algebra endowed with an order 3 automorphism. As a noteworthy example, the exceptional composition superalgebras will be obtained, in a systematic way, from the split octonion algebra.

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Lie algebras of vector fields on algebraic varieties

Vyacheslav Futorny[1]

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Derivations of the rings of functions on affine algebraic varieties is a source of (simple) infinite dimensional Lie algebras. Classical examples include the Virasoro and the Witt algebras, which correspond to the case of the affine space. Their representations are well understood. We will discuss the state of the art of the representation theory of derivation algebras in the case of arbitrary varieties, which is being developed rapidly and has many interesting unusual features.

Non-Associative Algebras and Related Topics II
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Coimbra, July 18–22, 2022

Gradings on simple algebras

Mikhail Kochetov[1]

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E-mail: mikhail@mun.ca

In the past two decades there has been considerable interest in classifying gradings by arbitrary groups on algebras of different varieties including associative, associative with involution, Lie and Jordan. Of particular importance are the so-called *fine gradings* (that is, those that do not admit a proper refinement), because any grading on a finite-dimensional algebra A can be obtained from them via a group homomorphism (although not in a unique way). If the ground field F is algebraically closed and of characteristic 0, then the classification of fine abelian group gradings on A up to equivalence is the same as the classification of maximal quasitori in the algebraic group $\mathrm{Aut}_F(A)$ up to conjugation. In particular, fine gradings have been classified for all finite-dimensional simple complex Lie and Jordan algebras by the efforts of many authors. In the case of classical Lie algebras (except D_4), the problem can be "transferred" to simple associative algebras with involution [1], and this was used by A. Elduque in [2] to complete the classification of fine gradings with the help of graded sesquilinear forms over associative graded-division algebras.

In this talk I will review some of these classification results and focus on the recent joint work with A. Elduque and A. Rodrigo-Escudero [3] where we simplify and extend the approach of [2] to an arbitrary ground field F, which allows us to classify fine gradings on simple associative algebras with involution over real closed fields and, as a consequence, on the real forms of classical simple complex Lie algebras.

Keywords: Graded algebra, graded module, classical simple Lie algebra

Mathematics Subject Classification 2020: 17B70, 16W50, 16W10

References

[1] Y. Bahturin, I. Shestakov, M. Zaicev, Gradings on simple Jordan and Lie algebras, *J. Algebra* **283** (2005), 849–868.

[2] A. Elduque, Fine gradings on simple classical Lie algebras, *J. Algebra* **324** (12) (2010), 3532–3571.

[3] A. Elduque, M. Kochetov and A. Rodrigo-Escudero, Gradings on associative algebras with involution and real forms of classical simple Lie algebras, *J. Algebra* **590** (2022), 61–138.

Non-Associative Algebras and Related Topics II
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Coimbra, July 18–22, 2022

On finitely presented (super)algebras

Consuelo Martínez[1]

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Superconformal algebras are graded superalgebras that are extensions of the (centerless) Virasoro algebra . The known superconformal algebras belong to one of the families $W(n:1)$, $S^{\alpha}(n:1)$, $K(n:1)$ or are the Cheng-Kac superalgebra $CK(6)$. The conjecture formulated by V. Kac and J. Van de Leur, see [1] is that there are no more superconformal algebras. The conjecture is still open, but several facts induce to believe that it is true (see [2–4]).

In this talk we will consider the following question:

Are these superalgebras finitely presented?

The afirmative answer was known for Neveu-Schwarz and Ramond algebras (see [5]).

But, in fact, we are interested in a wider class of superalgebras that correspond to arbitrary affine algebraic varieties.

The talk is based on the joint paper with Efim Zelmanov and David Zhang.

Keywords: superconformal algebras, finitely presented algebras

Mathematics Subject Classification 2020: 17B65, 17B68

References

[1] V. Kac and J.W.van de Leur, On classification of superconformal algebras, String'88, 77-106, World Sci. Pub. Teaneck NJ (1989).

[2] O. Mathieu, Classification of simple graded Lie algebras of finite growth, *Invent. Math.* **108**(1992), 455-519.

[3] V. Kac, C. Martinez and E. Zelmanov, Graded simple Jordan superalgebras of growth one, *Mem. Amer. Math. Soc.* **711** (2001).

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[5] D.B: Fairlie, J. Nuyts and C.K. Zachor, A presentation for the Virasoro and super-Virasoroalgebra, *Comm. Math. Physics* **117** (1988), no. 4, 595-614.

Non-Associative Algebras and Related Topics II
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Non-matrix varieties and coordinatization theorems for nonassociative algebras

Ivan Shestakov [1]

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The talk consists of two parts. The first part is devoted to varieties that do not contain matrix algebras. These varieties were introduced and first studied for associative algebras by V.N.Latyshev in [1]. Various characterizations of non-matrix varieties were obtained in [2]. We define non-matrix varieties for some classes of nonassociatgive algebras and obtain their characterizations generalizing the results of [2].

The second part of the talk is devoted to the coordinatization theorems for alternative and Jordan algebras, containing the matrix algebra of order 2 and the symmetric matrix algebra of order 2, correspondingly, with the same unit. Besides, we consider the coordinatization theorem for octonions in the class of right alternative algebras.

Keywords: non-matrix variety, coordinatization theorem, alternative algebra, Jordan algebra

Mathematics Subject Classification 2020: 17D05, 17C10, 17A15, 17A75, 16R10, 16R40

References

[1] LATYSHEV, V. N. The complexity of non-matrix varieties of associative algebras. I, II. *Algebra i Logika* **16**, no. 2 (1977), p. 149–183, 184–199.

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NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

On the growth of algebras

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We will discuss growth functions of infinite dimensional algebras, especially nil algebras. The talk is based on joint works with Jason Bell, [1], Be'eri Greenfeld, [2].

Keywords: growth, algebras

Mathematics Subject Classification 2020: 16P90,20M25

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6 Contributed Talks

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

On rigid 3-dimensional Hom-Lie algebras

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In this work we obtain all rigid complex 3-dimensional multiplicative Hom-Lie algebras. This is done by studying all deformations of multiplicative Hom-Lie algebras which are also Lie algebras. As a byproduct, we obtain the well-known classification of 3-dimensional multiplicative (non-Lie) Hom- Lie algebras. [1].

Keywords: Hom-Lie algebras, Rigidity, Cohomology, Deformations

Mathematics Subject Classification 2020: 17B56, 17B60, 17B99

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Quadratic Jordan algebras generated by two idempotents

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We will talk on the problem of extending to the quadratic case Rowen and Segev's results about Jordan algebras generated by two idempotents [1].

Keywords: Quadratic Jordan algebra, Idempotent

Mathematics Subject Classification 2020: 17C27, 17C10

References

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NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

2-Local and local derivations and automorphisms of Lie algebras

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Given an algebra $\mathcal{A}$, a linear mapping $T : \mathcal{A} \to \mathcal{A}$ is called an automorphism (respectively a *derivation*) if $T(ab) = T(a)T(b)$ (respectively, $T(ab) = T(a)b + aT(b)$) for all $a, b \in \mathcal{A}$.

A linear operator $\Delta : \mathcal{A} \to \mathcal{A}$ is called a *local automorphism* (respectively, *a local derivation*) if for every x in $\mathcal{A}$ there exists an automorphism α_x (respectively, a derivation d_x) on $\mathcal{A}$ depending on x, such that $\Delta(x) = \alpha_x(x)$ (respectively, $\Delta(x) = d_x(x)$).

A mapping $\Delta : \mathcal{A} \to \mathcal{A}$ (not linear in general) is called a 2-*local automorphism* (respectively, a 2-*local derivation*) on $\mathcal{A}$, if for every $x, y \in \mathcal{A}$, there exists an automorphism $\alpha_{x,y} : \mathcal{A} \to \mathcal{A}$ (respectively, a derivation $d_{x,y}$ on $\mathcal{A}$) such that $\Delta(x) = \alpha_{x,y}(x)$ and $\Delta(y) = \alpha_{x,y}(y)$ (respectively, $\Delta(x) = d_{x,y}(x)$ and $\Delta(y) = d_{x,y}(y)$).

The main problems concerning the above notions are to find conditions under which every local (or 2-local) automorphism or derivation automatically becomes an automorphism (respectively, a derivation), and also to present examples of algebras with local and 2-local automorphisms (respectively, derivations) that are not automorphisms (respectively, derivations). This talk is devoted to survey of the results concerning the above problems in the case of finite dimensional Lie algebras. We start with 2-local derivations and automorphisms on finite dimensional semi-simple and nilpotent Lie algebras, and then pass to local derivations and automorphisms of these algebras.

Keywords: Lie algebras, local derivation, 2-local derivation, local automorphism, 2-local automorphism,

Mathematics Subject Classification 2020: 16W25, 16W10

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Non-associative skew Laurent polynomial rings and related rings

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In this talk, I will introduce non-associative generalizations of skew Laurent polynomial rings and related rings, such as skew power series rings and skew Laurent series rings. The focus will mainly be on the former rings and their ideals, but we will also see how the famous Hilbert's basis theorem can be extended for all these rings to the non-associative setting.

This is joint work with Johan Richter.

Keywords: non-associative Hilbert's basis theorem, non-associative skew Laurent polynomial rings, non-associative skew Laurent series rings, non-associative skew power series rings

Mathematics Subject Classification 2020: 16S34, 16S36, 17A99

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

A deformed periplectic Brauer category

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The symmetric group algebra, the Brauer algebra and the periplectic Brauer algebra can all be represented using diagrams. For each class of these diagram algebras we can construct a corresponding category. Its objects are the natural numbers and homomorphisms between m and n in this category are given by the appropriate (m, n)-diagrams. The diagram algebras themselves can then be recovered by the endomorphism algebras in this category.

For the symmetric group algebra and the Brauer algebra there exists deformations, namely the Iwahori-Hecke algebra and the Birman-Wenzl-Murakami algebra. These deformed algebras can also be represented using diagrams. However, until recently a deformation of the periplectic Brauer algebra was not known.

In this talk I will introduce a new monoidal supercategory using diagrams. The endomorphism algebras of this category recover the quantum periplectic Brauer algebra recently introduced in the setting of Schur-Weyl duality for the quantized enveloping superalgebra of type P by Ahmed, Grantcharov and Guay.

Keywords: Diagram algebras, Periplectic Brauer algebra, Schur-Weyl duality

Mathematics Subject Classification 2020: 17B37, 16G10, 17B10

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Quadratic structures on flat pseudo-Euclidean Lie algebras

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In this talk we will give examples of quadratic structures on flat pseudo-Euclidean Lie algebras. Then we will explain the structure of flat Lorentzian pseudo-Euclidean Lie algebras which have quadratic structures.

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Double extensions related to oscillator algebras

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According to [2], oscillator algebras $\mathfrak{d}_{2n+2}$ are the only real indecomposable and non simple Lie algebras endowed with an invariant metric of index 1. They are the Lie algebras of oscillator Lie groups. The first step in this series, the 4-dimensional algebra $\mathfrak{d}_2$, arises in the quantum mechanical description of a harmonic oscillator and it is the smallest nonabelian solvable metric Lie algebra. Following [3, Proposition 2.2] (see also [1, Lemme 4.2]), for a fixed natural n, the algebra $\mathfrak{d}_{2n+2}$ appears as a double extension of a Hilbert space (V, φ) by a bijective skew φ-linear map $d\colon V \to V$. The derived algebra $\mathfrak{d}_{2n+2}^2$ is the nilpotent radical of $\mathfrak{d}_{2n+2}$ and it is just a $(2n+1)$-dimensional algebra of Heisenberg type. So, oscillator algebras are quadratic solvable. Starting with a vector space V and a bilinear symmetric and non-degenerate form φ, the series $\mathfrak{d}_{2n+2}$ can be consider over any field of characteristic zero. In this talk, we will provide some structural features on oscillator algebras and also explore their possibilities of being extended to mixed quadratic Lie algebras.

Keywords: Lie algebra, quadratic, double extension

Mathematics Subject Classification 2020: 17B05, 17B40

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Gradings, identities, and images of multilinear polynomials on matrix algebras

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Let $f = f(x_1, \ldots, x_m)$ be a noncommutative polynomial over a field K. If A is a K-algebra, such polynomial defines a map

$$\begin{array}{rccc} f: & A^m & \longrightarrow & A \\ & (a_1, \ldots, a_m) & \longmapsto & f(a_1, \ldots, a_m) \end{array}$$

which we will call the polynomial map defined by f and will denote simply by f.

A famous problem, known as *Lvov-Kaplansky's Conjecture* asks whether the image of a multilinear polynomial f on $M_n(K)$ is a vector subspace, or, equivalently, the image of f is one of the following subspaces: $\{0\}$, K (the set of scalar matrices), $sl_n(K)$ (the set of traceless matrices) or $M_n(K)$. When K is an algebraically closed fied, such problem has a positive answer only for $n = 2$ or $m = 2$. There are some parcial answers for $n = 3$ and $m = 3$, but no complete solution is known.

Recently, many variations of such conjecture have been studied. For instance, images of multilinear polynomials on upper (and strictly upper) triangular matrices, images of polynomials on quaternion algebras and on Jordan algebras of a symmetric bilinear form have been completely described.

In this talk we intend to explore how gradings on $M_n(K)$ can be used to study images of (ordinary) multilinear polynomials and also polynomial identities on $M_n(K)$. Also, we intend to present some new results and conjectures about images of graded polynomials on $M_n(K)$, when it is endowed with some special kind of gradings.

Keywords: Gradings, Images of polynomials, Polynomial identities

Mathematics Subject Classification 2020: 16R10, 16R50

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Non-associative Frobenius algebras for simple algebraic groups

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Recently, Maurice Chayet and Skip Garibaldi have introduced a class of G-equivariant commutative non-associative Frobenius algebras, for each simple linear algebraic group G over an arbitrary field of large enough characteristic [1]. We are able to give an explicit description of these algebras for groups of type G_2 and F_4 in terms of the octonion algebras and the Albert algebras, respectively.

It had already been observed by Chayet and Garibaldi that the automorphism group for the algebras for type F_4 is equal to the group of type F_4 itself (and nothing more). Using our new description, we are able to show that the same result holds for type G_2.

Keywords: automorphism groups, exceptional groups

Mathematics Subject Classification 2020: 20F29, 20G41

References

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Weak Leibniz algebras

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An algebra A is called *weak-Leibniz*, if it satisfies the following identities

$$[a,b]c-2a(bc)+2b(ac)=0, \quad a[b,c]-2(ab)c+2(ac)b=0,$$

for any $a,b,c\in A$, where $[a,b]=ab-ba$.

Example. Any two-sided Leibniz algebra, in particular, any Lie algebra is weak Leibniz. Converse is not true. Note that any simple Leibniz algebra is Lie.

An algebra with two binary operations $A=(A,\circ,\bullet)$ is called *transposed Poisson* [1], if $(A,\circ)$ is a Lie, $(A,\bullet)$ is an associative commutative and associative algebra acts on Lie algebra as 1/2-derivation,

$$2\,u\bullet(a\circ b)=(u\bullet a)\circ b+a\circ(u\bullet b), \quad \forall u,a,b,c\in A.$$

An algebra A is called *transposed Poisson-admissble,* if $(A,\circ,\bullet)$ is transposed Poisson for $a\circ b=ab-ba, a\bullet b=ab+ba$.

Theorem 1. *An algebra $A=(A,\circ,\bullet)$ is transposed Poisson-admissible if and only if it is weak Leibniz under operation $ab=1/2(a\bullet b+a\circ b)$.*

Definition. Let us given a vector space L with one unary linear operation $f:A\to A, a\mapsto a^{\S}$ called *weight* and two bilinear operations $A\times A\to A, (a,b)\mapsto a\cdot b, (a,b)\mapsto a\bullet b$. These operations satisfy the following conditions.

I. $(L,\cdot)$ is left-Novikov, i.e., the following identities hold

$$(a\cdot b)\cdot c-(b\cdot a)\cdot c=a\cdot(b\cdot c)-b\cdot(a\cdot c),$$

$$(a\cdot b)\cdot c=(a\cdot c)\cdot b,$$

II. $(L,\bullet)$ is associative-commutative,

$$a\bullet(b\bullet c)=(a\bullet b)\bullet c,$$

$$a\bullet b=b\bullet a,$$

III. multiplications $\cdot$ and $\bullet$ are connected by the following identities

$$(a \bullet b) \cdot c = a \bullet (b \cdot c),$$

$$(a \cdot b) \bullet c - (b \cdot a) \bullet c = a \cdot (b \bullet c) - b \cdot (a \bullet c),$$

$$a \cdot (b \bullet c) - (a \cdot b) \bullet c - b \bullet (a \cdot c) = a^{\S} \bullet (b \bullet c)$$

Call such algebra *weighted Novikov-Poisson algebra.* If weight is trivial, $a^{\S} = 0$ for any $a \in L$, then we obtain Novikov-Poisson algebra.

Propostition. *Let $(A, \cdot, \bullet, \S)$ be weighted Novikov-Poisson algebra and $A_{u,v,w} = (A, \circ_{u,v}, \bullet_w)$ algebra with two binary operations $a \circ_{u,v} b = a \cdot_{u,v} b - b \cdot_{u,v} a$ and $a \bullet_w b = w \bullet (a \bullet b)$. Then for any invertible $w \in A$ the algebra $A_{u,v,w}$ is transposed Poisson.*

We apply this propositiion to construct simple non-Lie weak Leibniz algebra. Let I be finite set of integers, $\epsilon_i \in K$, for any $i \in I$. Let $L(I, \epsilon)$ be infinite-dimensional algebra with base $\{e_i | i \in \mathbf{Z}\}$ and multiplication

$$e_i e_j = (i - j) e_{i+j} + \sum_{k \in I} \epsilon_k e_{i+j+k}.$$

Let

$$h(t_1, t_2, t_3, t_4, t_5) =$$

$$(((t_1t_3)t_4)t_5)t_2 - (((t_1t_3)t_5)t_4)t_2 - (((t_1t_4)t_3)t_5)t_2 + (((t_1t_4)t_5)t_3)t_2 + (((t_1t_5)t_3)t_4)t_2 - (((t_1t_5)t_4)t_3)t_2 +$$

$$2\{-(((t_1t_3)t_2)t_4)t_5 + (((t_1t_3)t_2)t_5)t_4 + (((t_1t_4)t_2)t_3)t_5 - (((t_1t_4)t_2)t_5)t_3 - (((t_1t_5)t_2)t_3)t_4 + (((t_1t_5)t_2)t_4)t_3\}.$$

Theorem 2. *The algebra $L(I, \epsilon)$ is simple, weak Leibniz and is not Lie. It satisfies the following identity of degree* 5

$$h(a, b, c, u, v) = 0,$$

for any $a, b, c, u, v \in L(I, \epsilon)$. This identity is not consequence of weak Leibniz identities.

Theorem 3. *Weak Leibniz operad is self-dual, i.e., its Kooszul dual is equivalent to weak Leibniz operad. Weak Leibniz operad is not Koszul. Any weak Leibniz algebra is associative-admissible, Lie-admissible and* 1*-Alia, but not Poisson-admissible.*

Keywords: Leibniz algebras, Poisson algeras, Operads, Koszul duality

Mathematics Subject Classification 2020: 17B63,17D25,16S37

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Differential graded Lie algebras in Hodge theory

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Let U be a smooth connected complex algebraic variety, and let $f : U \to \mathbb{C}^*$ be an algebraic morphism. The Alexander modules of (U, f) are the homology groups of a covering space of U induced by f, and this covering space is not an algebraic variety in general. Deligne [1, 2] endowed the cohomology groups of complex algebraic varieties with a canonical and functorial mixed Hodge structure (MHS). In joint work with Geske, Herradón Cueto, Maxim and Wang ([3]), we constructed a MHS on the torsion part of Alexander modules, extending Deligne's construction. In this talk, we will give an overview of Hodge theory, and explain how graded differentiable Lie algebras provide a convenient framework for the extension of Deligne's theory to covering spaces of algebraic varieties. This is based on ongoing work with Moisés Herradón Cueto.

Keywords: infinite cyclic cover, Alexander module, mixed Hodge structure, thickened complex, differential graded Lie algebras

Mathematics Subject Classification 2020: 14C30, 14F40, 14F45, 32S20, 32S35, 32S40, 55N30

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NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Gradings induced by nilpotent elements

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An element a is nilpotent last-regular if it is nilpotent and its last nonzero power is von Neumann regular. In this paper we show that any nilpotent last-regular element a in an associative algebra R over a ring of scalars Φ gives rise to a complete system of orthogonal idempotents that induces a finite $\mathbb{Z}$-grading on R; we also show that such element gives rise to an $\mathfrak{sl}_2$-triple in R with semisimple adjoint map ad_h, and that the grading of R with respect to the complete system of orthogonal idempotents is a refinement of the Φ-grading induced by the eigenspaces of ad_h. These results can be adapted to nilpotent elements a with all their powers von Neumann regular, in which case the element a can be completed to an $\mathfrak{sl}_2$-triple and a is homogeneous of degree 2 both in the $\mathbb{Z}$-grading of R and in the Φ-grading given by the eigenspaces of ad_h.

Conversely, when there are enough invertible elements in Φ, we will show that if a nilpotent element a can be completed to an $\mathfrak{sl}_2$-triple, then all powers a^k are von Neumann regular.

Keywords: von Neumann regular, nilpotent element, grading, $\mathfrak{sl}_2$-triple

Mathematics Subject Classification 2020: 16E50, 16W50

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Hom-associative algebras and divisibility

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Transitive character of divisibility is a fundamental feature of associative algebras. In general, this property relies on one factor being a central element of the algebra.

A similar feature can be found in Hom-associative algebras. Considering the twisting map it is possible to establish analogous properties related to divisibility chains and zero divisors.

We extend these ideas to various known and new classes of non-associative Hom-algebras.

Keywords: Hom-algebras, Divisibility

Mathematics Subject Classification 2020: 17A30, 17A36, 17B61

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Two characterisations of Lie algebras

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The variety of Lie algebras is usually one of the central examples in the non-associative algebras world. Many properties, studied from a universal algebra point of view, were firstly introduced to Lie algebras and then generalised to other different structures. One of the many aims of categorical algebra is to give general definitions that can help to understand these notions and their generalisations.

The aim of this talk is to discuss two categorical algebraic ideas, *representability of actions* and the existence of *algebraic exponents*. They are both interesting in the following way: we can characterise the variety of Lie algebras amongst all varieties of non-associative algebras over an infinite field. Moreover, these characterisations are *categorical*; in the sense that no elements are used, only morphisms and universal properties.

We will discuss the algebraic meanings of these properties, the computational methods used to obtain the characterisations, and what can be done beyond them.

Joint work with Matsvei Tsishyn, Corentin Vienne and Tim Van der Linden.

Keywords: Lie algebras, action representability, algebraic exponents

Mathematics Subject Classification 2020: 17A36

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Wedderburn principal theorem for Jordan superalgebras type $\mathfrak{Kan}(2)$

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We consider a finite dimensional Jordan superalgebra $\mathcal{A}$ over an algebraically closed field $\mathbb{F}$ with solvable radical $\mathcal{N}$ such that $\mathcal{N}^2 = 0$ and the quotient superalgebra $\mathcal{A}/\mathcal{N}$ is isomorphic to simple Jordan superalgebra of Grassmann Poisson bracket. We use some preliminary results gave by the first author in [1], where he proved that an analogue to WPT is valid under some conditions. In particular he proved that if $\mathcal{N}$ is a solvable radical of a finite dimensional Jordan superalgebra over an algebraically closed field $\mathcal{A}$ and $\mathcal{N}^2 = 0$, then we can assume that $\mathcal{A}/\mathcal{N}$ is a simple Jordan superalgebra with unit $\mathcal{J}$ or is a hull Jordan superalgebra $\mathcal{H}$. Hence, he proved that it is sufficient to reduce the problem to case when $\mathcal{N}$ is isomorphic with an irreducible $\mathcal{J}$-Jordan superbimodule.

In this talk, we consider the simple Jordan superalgebra $\mathfrak{Kan}(2)$, and we use the classifications of irreducible $\mathfrak{Kan}(2)$-Jordan superbimodules gave by Folleco and Shestakov [2].

Finally we determined some conditions on the split extension $\mathfrak{Kan}(2) \oplus V(v, \alpha)$ and $\mathfrak{Kan}(2) \oplus V(v, \alpha)^{\mathrm{op}}$; in both cases we concluded that an analogue to WPT is valid.

Keywords: Wedderburn principal theorem, second cohomology group, decompositions theorem, Jordan superalgebras.

Mathematics Subject Classification 2020: 17A15, 17A70, 17A50

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Low dimensional commutative power-associative superalgebras†

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The aim of this work is to provide a concrete list of non-isomorphic commutative power-associative superalgebras up to dimension 4 over an algebraically closed field of characteristic prime to 30. As a byproduct, we exhibit an example of a simple non-Jordan power-associative superalgebra whose even part is not semisimple. [1].

Keywords: Power-associative algebras, Commutative power-associative superalgebras

Mathematics Subject Classification 2020: 17A70, 17A05, 17C27

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Abelian Tits sets and Jordan algebras

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Moufang sets were introduced by Jacques Tits as rank 1 buildings. They are certain 2 transitive permutation groups and axiomatize the action of a semisimple algebraic group on its Borel subgroups. The (Moufang) buildings of rank at least 2 have been classified, the classification of the Moufang sets however, is still an openstanding problem.

A special class of Moufang sets are those with abelian root groups. Every known (proper) abelian Moufang set corresponds in a canonical way to a Jordan division algebra and visa versa, and it has been conjectured that these are all. In this talk, we discuss a generalization of Moufang sets, namely Tits sets. It turns out that abelian Tits sets that are not Moufang (under some natural conditions) correspond to the simple Jordan algebras of finite capacity that are not division.

Keywords: Moufang sets, Jordan algebras, Buildings

Mathematics Subject Classification 2020: 17A15, 20E42, 20G15

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
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New constructions of exceptional Lie superalgebras using tensor categories

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Using tensor categories, we present new constructions of several of the exceptional simple Lie superalgebras with integer Cartan matrix in characteristic $p = 3$ and $p = 5$, which were classified in [1]. These include the Elduque and Cunha Lie superalgebras. Specifically, let $\boldsymbol{\alpha}_p$ denote the kernel of the Frobenius endomorphism on the additive group scheme $\mathbb{G}_a$ over an algebraically closed field of characteristic p. The Verlinde category Ver_p is the *semisimplification* of the representation category Rep $\boldsymbol{\alpha}_p$, and Ver_p contains the category of super vector spaces as a full subcategory. Each exceptional Lie superalgebra we construct is realized as the image of an exceptional Lie algebra equipped with a nilpotent derivation of order at most p under the semisimplification functor from Rep $\boldsymbol{\alpha}_p$ to Ver_p.

Keywords: modular Lie superalgebras, symmetric tensor categories

Mathematics Subject Classification 2020: 17B, 18M20

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Free Jordan algebras

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We will discuss a conjecture for the character of the homogenous components of the free Jordan algebra on d generators as a $GL(d)$-module. This is joint work with Olivier Mathieu, [1].

Keywords: Free Jordan algebra, Tits-Kantor-Koecher construction

Mathematics Subject Classification 2020: 17A50, 17C50

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Universal central extensions of compatible Leibniz algebras[‡]

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Two algebraic structures of the same type (i.e. both are associative algebras, Lie algebras, etc.) $(V, \star)$ and $(V, \star')$ with the same underlying vector space, but with different operations, are said to be compatible if any linear combination of both operations $\lambda\cdot\star+\mu\cdot\star'$ endows V with the same algebraic structure as the first ones.

For instance, a compatible Lie algebra [5] is a vector space $\mathfrak{g}$ endowed with two skew-symmetric bracket operations $[-,-]$ and $(-,-)$ such that the following identity holds for any $x, y, z \in \mathfrak{g}$

$$([x,y],z)+([y,z],x)+([z,x],y)+[(x,y),z]+[(y,z),x]+[(z,x),y]=0. \tag{1}$$

This compatible Jacobi identity (1) is derived by imposing on the bracket $\{x,y\} = \lambda_1[x,y]+\lambda_2(x,y)$, for any $\lambda_1, \lambda_2 \in \mathbb{K}$, the skew-symmetry and Jacobi identity, bearing in mind that $(\mathfrak{g},[-,-])$ and $(\mathfrak{g},(-,-))$ are Lie algebras.

Compatible algebraic structures are considered in many fields of mathematics and mathematical physics. For instance, compatible Lie algebras appear in the context of integrable Hamiltonian equations [4], in the context of the Yang-Baxter equation and principal chiral field [2], or in the context of elliptic theta functions [3].

In this talk, we focus on the structure of compatible Leibniz algebra. So $(\mathfrak{g},[-,-], (-,-))$ is a compatible Leibniz algebra if and only if the following condition holds for any $x, y, z \in \mathfrak{g}$:

$$[x,(y,z)]-[(x,y),z]+[(x,z),y]+(x,[y,z])-([x,y],z)+([x,z],y)=0, \tag{2}$$

provided that $(\mathfrak{g},[-,-])$ and $(\mathfrak{g},(-,-))$ are Leibniz algebras.

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First of all, we establish the relations between compatible Lie, Leibniz, Associative algebras and Dialgebras showing the commutativity of the following diagram

$$\begin{tikzcd} \mathsf{CDias} \arrow[r] & \mathsf{CLeib} \\ \mathsf{CAss} \arrow[r] \arrow[u] & \mathsf{CLie}, \arrow[u] \end{tikzcd}$$

where CDias, CAss, CLie and CLeib denote the categories of compatible dialgebras, compatible associative algebras, compatible Lie algebras and compatible Leibniz algebras, respectively.

Then we construct a homology with trivial coefficients for compatible Leibniz algebras and develop a theory of universal central extensions of compatible Leibniz algebras. The main result states, "A compatible Leibniz algebra admits universal central extension if and only if it is perfect". Moreover, the kernel of the universal central extension is the first homology with trivial coefficients of the perfect compatible Leibniz algebra.

Another important property that allows characterizing universal central extensions is the so-called UCE condition, established in [1]. Namely, the composition of central extensions (the middle term in one of them must be perfect) is also a universal central extension.

We try to prove this fact by mimicking the proof for Leibniz algebras, but it does not hold. It only holds if additional requirements are assumed. So we conjecture this property is not valid for compatible Leibniz algebras.

Keywords: Compatible Leibniz algebra, Homology, Universal central extension

Mathematics Subject Classification 2020: 17A32

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Two-step nilpotent Leibniz algebras

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Leibniz algebras were first introduced by J.-L. Loday in [2] as a non-antisymmetric version of Lie algebras, and many results of Lie algebras have been extended to Leibniz algebras. Earlier, such algebraic structures had been considered by A. Blokh, who called them D-algebras. Nowdays Leibniz algebras play a significant role in different areas of mathematics and physics.

In this talk we give the classification of two-step nilpotent Leibniz algebras over a field $\mathbb{F}$, with $char(\mathbb{F}) \neq 2$, in terms of Kronecker modules associated with pairs of bilinear forms. We show that there are only three classes of nilpotent Leibniz algebras with one-dimensional commutator ideal, which we call the *Heisenberg Leibniz algebras* $\mathfrak{l}_{2n+1}^A$, parametrized by the dimension $2n+1$ and a $n \times n$ matrix A in canonical form, the *Kronecker Leibniz algebras* $\mathfrak{k}_n$ and the *Dieudonné Leibniz algebras* $\mathfrak{d}_n$, both parametrized by their dimension only.

Moreover, using the Leibniz algebras / Lie local racks correspondence (see [4] and [5]), we show that nilpotent real Leibniz algebras have always a *global* integration. As an example, we integrate the indecomposable nilpotent real Leibniz algebras with one-dimensional commutator ideal.

Finally we show that every Lie quandle integrating a Leibniz algebra is induced by the conjugation of a Lie group and the Leibniz algebra is the Lie algebra of that Lie group.

This is joint work with Gianmarco La Rosa (*University of Palermo*).

Keywords: Leibniz algebras, Nilpotent Leibniz algebras, Lie racks, Coquegigrue problem.

Mathematics Subject Classification 2020: 17A32, 22A30, 20M99.

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
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Differential identities and polynomial growth of the codimensions

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Let L be a Lie algebra acting by derivations on a finite dimensional associative F-algebra A, where F is a field of characteristic zero. Recall that a derivation $\delta : A \to A$ is a linear map such that

$$\delta(ab) = \delta(a)b + a\delta(b),$$

for all $a, b \in A$. It also turns out that $\delta(\alpha) = 0$ for all $\alpha \in F$. Roughly speaking, a derivation on A is the "algebraic" generalization of the well-known calculus-like derivative. Such action can be naturally extended to the universal enveloping algebra $U(L)$ of L and in this case we say that A is an algebra with derivation or simply an L-algebra.

Now let $F\langle X|L\rangle$ be the free associative algebra with derivation, freely generated by the countable set of variables $X = \{x_1, x_2, \ldots\}$ on F. A polynomial $f(x_1^{h_1}, \ldots, x_n^{h_n}) \in F\langle X|L\rangle$, where $h_1, \ldots, h_n \in U(L)$, is called L-polynomial identity or differential identity of the L-algebra A, if $f(a_1^{h_1}, \ldots, a_n^{h_n}) = 0$ for all $a_1, \ldots, a_n \in A$. We also denote by $\mathrm{Id}^L(A) = \{f \in F\langle X|L\rangle \mid f \equiv 0 \text{ on } A\}$ the T_L-ideal of L-identities of A.

As in the ordinary case, in characteristic zero, every L-identity is equivalent to a system of multilinear ones. We denote by

$$P_n^L = \mathrm{span}\{x_{\sigma(1)}^{d_{i_1}} \cdots x_{\sigma(n)}^{d_{i_n}} \mid \sigma \in S_n, d_{i_k} \in U(L)\}$$

the vector space of multilinear differential polynomials in the variables $x_1, \ldots, x_n$, $n \geq 1$. Since $\mathrm{Id}^L(A)$ is generated, as T_L-ideal, by the multilinear L-polynomials it contains, the study of $\mathrm{Id}^L(A)$ is equivalent to the study of $P_n^L \cap \mathrm{Id}^L(A)$ for all $n \geq 1$. In case $U(L)$ acts on A as a suitable finite dimensional subalgbera of the endomorphism ring of A, then P_n^L is finite dimensional and we denote by

$$c_n^L(A) = \dim_F \frac{P_n^L}{P_n^L \cap \mathrm{Id}^L(A)}, \quad n \geq 1,$$

the nth differential codimension of A or the nth L-codimension of A.

Generalizing the previous definition in case of L-varieties of finite dimensional associative L-algebras, it was proved that $c_n^L(\mathrm{var}^L(A))$ has almost polynomial growth in case $A = UT_2$ or $A = UT_2^\varepsilon$, where UT_2 is the 2×2 upper triangular matrix algebra with trivial L-action, and UT_2^ε is the corresponding L-algebra where L acts on UT_2 as the

1-dimensional Lie algebra spanned by the inner derivation ε induced by e_{11}. Recall that almost polynomial growth means that $c_n^L(\mathrm{var}^L(A))$ grows exponentially but the codimension sequence of any proper L-subvariety grows polynomially.

In this talk we present the results of [1] in which all the L-subvarieties of $\mathrm{var}^L(UT_2)$ and $\mathrm{var}^L(UT_2^{\varepsilon})$ were classified by giving a complete list of finite dimensional L-algebras generating them.

Keywords: Polynomial identity, differential identity, polynomial growth, codimension

Mathematics Subject Classification 2020: Primary 16R10, 16R50; Secondary 16W25, 16P90

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Complex cyclic Leibniz superalgebras

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Since Loday introduction of Leibniz algebras as a generalisation of Lie algebras, many results of the theory of Lie algebras have been extended to Leibniz algebras. Cyclic Leibniz algebras, which are generated by one element, have no equivalent into Lie algebras, though. This fact provides cyclic Leibniz algebras with important properties. Throughout the present work we extend the concept of cyclic to Leibniz superalgebras, obtaining then the definition, as long as the description and classification of finite-dimensional complex cyclic Leibniz superalgebras. Furthermore, we prove that any cyclic Leibniz superalgebra can be obtained by means of infinitesimal deformations of the null-filiform Leibniz superalgebra. We also obtain a description of irreducible components in the variety of Leibniz algebras and superalgebras. [1].

Keywords: Leibniz superalgebra; cyclic Leibniz superalgebra; null-filiform superalgebra; infinitesimal deformation; irreducible component.

Mathematics Subject Classification 2020: 17A32; 17A70; 17B30; 13D10.

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NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

On some properties of generalized Lie-derivations of Leibniz algebras

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The Liezation functor $(-)_{\mathsf{Lie}} : \mathsf{Leib} \to \mathsf{Lie}$, assigns to a Leibniz algebra $\mathfrak{g}$ the Lie algebra $\mathfrak{g}_{\mathrm{Lie}} = \frac{\mathfrak{g}}{<\{[x,x]:x\in\mathfrak{g}\}>}$ (Leib denotes the category of Leibniz algebras and Lie denotes the category of Lie algebras) and it is left adjoint to the inclusion functor $inc : \mathsf{Lie} \to \mathsf{Leib}$ [6]. This environment fits in the framework of central extension and commutators in semi-abelian categories with respect to a Birkhoff subcategory [4,5], where classical or absolute notions are relative to the abelianization functor. All this information can be summarized in the following diagram

$$\begin{tikzcd} \mathsf{Leib} \arrow[rr, shift left=1.5, "(-)_{\mathsf{Lie}}"] \arrow[rr, phantom, "\perp"] \arrow[dr, "\mathsf{ab}"'] & & \mathsf{Lie} \arrow[ll, shift left=1.5, "inc"] \arrow[dl, "\mathsf{ab}"] \\ & \mathsf{Vect} & \end{tikzcd}$$

In [3] classical properties of Leibniz algebras (properties relative to the abelianization functor) were adapted to the relative setting (with respect to the Liezation functor); in general, absolute properties have the corresponding relative ones, but not all absolute properties immediately hold in the relative case, so new requirements are needed. The study of relative properties helps to understand the structure of Leibniz algebra.

Following this line of research, in the papers [1,2] was conducted an analysis of central derivations of Leibniz algebras relative to the Liezation functor, called as Lie-derivations. They were used to characterize Lie-stem Leibniz algebras. Also the interplay between Lie-central derivations and Lie-centroids was studied.

In this talk we present an overview of these results and we analyze some new properties concerning Lie-derivations, Lie-central derivations, almost inner Lie-derivations, ID-Lie-derivation, ID_*-Lie-derivation and Lie-centroid of Leibniz algebras.

In summary, a *Leibniz algebra* [5] is a vector space $\mathfrak{g}$ equipped with a linear map $[-,-] : \mathfrak{g}\otimes\mathfrak{g} \to \mathfrak{g}$ satisfying the Leibniz identity $[x,[y,z]] = [[x,y],z] - [[x,z],y], x,y,z \in \mathfrak{g}$.

We define the bracket $[-,-]_{lie} : \mathfrak{g} \to \mathfrak{g}$, by $[x,y]_{lie} = [x,y] + [y,x]$, for $x, y \in \mathfrak{g}$. The Lie-center of $\mathfrak{g}$ is the two-sided ideal $Z_{\mathsf{Lie}}(\mathfrak{g}) = \{x \in \mathfrak{g} \mid [x,y]_{lie} = 0, \forall y \in \mathfrak{g}\}$ [3]. From [1] we have the following notions: a Lie-derivation is the linear map, $d : \mathfrak{g} \to \mathfrak{g}$ such that, $d([x,y]_{lie}) = [d(x),y]_{lie} + [x,d(y)]_{lie}$, for all $x, y \in \mathfrak{g}$, which itself forms a Lie algebra with respect to the commutator of linear transformations denoted by $\mathsf{Der}^{\mathsf{Lie}}(\mathfrak{g})$. A Lie-derivation is called a Lie-central derivation if its image is contained in the Lie-center of $\mathfrak{g}$. The set of all Lie-central derivation is denoted by $\mathsf{Der}_z^{\mathsf{Lie}}(\mathfrak{g})$. A Lie-derivation d is called an almost inner Lie-derivation if $d(x) \in [x, \mathfrak{g}]_{\mathsf{Lie}}$, for all $x \in \mathfrak{g}$. The set of all almost inner Lie-derivation is denoted by $\mathsf{Der}_c^{\mathsf{Lie}}(\mathfrak{g})$. A Lie-derivation $d : \mathfrak{g} \to \mathfrak{g}$ is said to be an ID-Lie-derivation if $d(\mathfrak{g}) \subseteq \gamma_2^{\mathsf{Lie}}(\mathfrak{g})$. The set of all ID-Lie-derivations of $\mathfrak{g}$ is denoted by $\mathsf{ID}^{\mathsf{Lie}}(\mathfrak{g})$. An ID-Lie-derivation $d : \mathfrak{g} \to \mathfrak{g}$ is said to be ID_*-Lie-derivation if d vanishes on the Lie-central elements of $\mathfrak{g}$. The set of all ID_*-Lie-derivations of $\mathfrak{g}$ is denoted by $\mathsf{ID}_*^{\mathsf{Lie}}(\mathfrak{g})$. A Lie-centroid of a Leibniz algebra $\mathfrak{g}$ is the set of all linear maps $d : \mathfrak{g} \to \mathfrak{g}$ such that $d([x,y])_{lie} = [d(x),y]_{lie} = [x,d(y)]_{lie}$ for all $x, y \in \mathfrak{g}$. We denote this set by $\Gamma^{\mathsf{Lie}}(\mathfrak{g})$.

Our main results address the following aspects:

1. We compute and analyze properties of $\mathsf{Der}^{\mathsf{Lie}}(\mathfrak{g})$, $\mathsf{Der}_z^{\mathsf{Lie}}(\mathfrak{g})$, $\mathsf{Der}_c^{\mathsf{Lie}}(\mathfrak{g})$, $\mathsf{ID}^{\mathsf{Lie}}(\mathfrak{g})$, $\mathsf{ID}_*^{\mathsf{Lie}}(\mathfrak{g})$ and $\Gamma^{\mathsf{Lie}}(\mathfrak{g})$ when $\mathfrak{g}$ is a low-dimensional Leibniz algebra.

2. Let $\mathfrak{g}_1$ and $\mathfrak{g}_2$ be two finite dimensional Leibniz algebras such that they have no non-trivial common direct factor and $\mathfrak{g} = \mathfrak{g}_1 \oplus \mathfrak{g}_2$. We establish the relation between the generalized Lie-derivations of $\mathfrak{g}$ in terms of the corresponding generalized Lie-derivations of $\mathfrak{g}_1$ and $\mathfrak{g}_2$.

Keywords: Lie-derivation, Lie-central derivation, almost inner Lie-derivation, ID-Lie-derivation, ID_*-Lie-derivation, Lie-centroid

Mathematics Subject Classification 2020: 17A32, 17A36, 17B40

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
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On the subalgebra lattice of a restricted Lie algebra

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The study of the subalgebra lattice of a finite-dimensional Lie algebra was popular in the last decades of the past century, but interest afterwards waned given the few algebras that satisfied the strong conditions under investigation. However, the lattice of restricted subalgebras of a restricted Lie algebra is fundamentally different: the fact that there are one-dimensional subalgebras which are not restricted results in some of the lattice conditions being weaker than in the non-restricted case, yielding more interesting conclusions. In this talk, based on [1], we will address the algebras in which this lattice is dually atomistic, lower or upper semimodular, or in which every restricted subalgebra is a quasi-ideal.

Keywords: Restricted Lie algebra, restricted subalgebra, dually atomistic, restricted quasi-ideal, lower semimodular, upper semimodular

Mathematics Subject Classification 2020: 17B50, 17B05, 06C10

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NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Phoenix restricted Lie algebras

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Different versions of BURNSIDE PROBLEM ask what one can say about finitely generated periodic groups under additional assumptions. For associative algebras, KUROSH type problems yields similar questions about properties of finitely generated nil (more generally, algebraic) algebras. Similarly, one considers finitely generated restricted Lie algebras with a nil p-mapping. Now we study an oscillating intermediate growth in the class of NIL restricted Lie algebras.

Namely, for any field of positive characteristic, we construct a family of 3-generated restricted Lie algebras of intermediate oscillating growth [1, 2]. We call them *Phoenix algebras*, because of the following properties.

1. For infinitely many periods of time the algebra is "almost dying" by having a *quasi-linear* growth, namely the lower Gelfand-Kirillov dimension is one, more precisely, the growth is of type $n\big(\underbrace{\ln\cdots\ln}_{q \text{ times}} n\big)^{\kappa}$, where $q \in \mathbb{N}$, $\kappa > 0$ are constants.

2. On the other hand, for infinitely many n the growth function has a rather fast intermediate behaviour of type $\exp(n/(\ln n)^{\lambda})$, λ being a constant determined by characteristic, for such periods the algebra is "resuscitating".

3. Moreover, the growth function is bounded and oscillating between these two types of behaviour.

4. These restricted Lie algebras have a nil p-mapping.

Keywords: restricted Lie algebras, growth, nil-algebras, Kurosh problem

Mathematics Subject Classification 2020: 16P90, 16N40, 17B50, 17B66

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NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Second cohomology group for finite-dimensional simple Jordan superalgebra $\mathcal{D}_t$ with $t \neq 0$

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We consider the finite-dimensional simple Jordan Superalgebra $\mathcal{D}_t$, $t \neq 0$. By [1], first author showed that $\mathcal{D}_t$, $t \neq 0$, does not satisfy Wedderburn Principal Theorem, therefore the second cohomology group (SCG) of the Jordan superalgebra $\mathcal{D}_t$, $t \neq 0$, over an algebraically closed field $\mathbb{F}$ of characteristic zero is not trivial. We calculate the SCG for the Jordan superalgebra $\mathcal{D}_t$, $t \neq 0$ using the coefficients which appear in the regular superbimodule Reg$\mathcal{D}_t$.

First, to calculate the SCG of a Jordan superalgebra we use split-null extension of the Jordan superalgebra and the Jordan superalgebra representation. Second, we prove conditions that satisfy the bilinear forms h that determine the SCG in Jordan superalgebras. We use these to calculate the SCG for the Jordan superalgebra $\mathcal{D}_t$, $t \neq 0$.

Finally, we prove that $\mathcal{H}^2(\mathcal{D}_t, \mathrm{Reg}\mathcal{D}_t) = 0 \dot{+} \mathbb{F}^2$, $t \neq 0$. This result is similar to the result of SCG for alternative superalgebra $\mathcal{M}_{1|1}(\mathbb{F})$ obtained by Pisarenko [3,4] (see, also, López-Díaz [5] (Theorem 2. (iii), p.261).

Keywords: Jordan superalgebra, second cohomology group, Wedderburn principal theorem, split null extension, regular superbimodule, decomposition theorem.

Mathematics Subject Classification 2020: 17A70, 17C70, 17A60

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NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Lattices of ideals in quadratic Lie algebras

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A quadratic Lie algebra is a Lie algebra with an invariant non-degenerate symmetric bilinear form. When we study the ideals of these algebras some important properties appears, for example, the orthogonal subspace of the descending central series is the ascending one, and viceversa. Even more, the lattice of ideals must be self-dual. This greatly reduces the shape of possible lattices of these algebras and also constrains their algebraic structure. In this talk we are going to see these properties, along with some constructions models to obtain quadratic Lie algebras.

Keywords: lattice of ideals, quadratic, Lie algebra

Mathematics Subject Classification 2020: 17B05, 17B40, 06D50

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Some functors preserving minimal projective resolutions

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We are interested in the construction of projective resolutions with good combinatorial properties for Weyl modules for the general linear group.

Let R be a commutative ring. Denote by $\mathfrak{U}_n^+(R)$ the Kostant form over R of the universal enveloping algebra of the Lie algebra of $n \times n$ complex nilpotent upper triangular matrices. In this talk I will explain the construction of functors that map (minimal) projective resolutions of the trivial rank-one $\mathfrak{U}_n^+(R)$-module to (minimal) projective resolutions of rank-one modules for the Borel-Schur algebra. From these, one can easily obtain projective resolutions of the Weyl modules for the Schur algebra and for the general linear group.

Keywords: Projective resolutions, Schur algebras

Mathematics Subject Classification 2020: 20G43,16W50

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

A characterization of the quaternions using commutators

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We will prove the following theorem.

THEOREM.
Let R be an associative ring with $\mathbf{1}$ which is not commutative such that

(i) A non-zero commutator in R is not a divisor of zero in R;

(ii) $(x, y)^2 \in C$, for all $x, y \in R$, where C is the center of R.

Then

1. R contains no divisors of zero.
2. If, in addition, the characteristic of R is not 2, then the localization of R at C is a quaternion division algebra, whose center is the fraction field of C.

We note that if $x, y \in R$ are non-zero elements such that $xy = 0$, then we say that both x and y are zero divisors in R.

Keywords: quaternion algebra, zero divisor, commutator

Mathematics Subject Classification 2020: Primary: 12E15, Secondary: 11R52

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[KS4] E. Kleinfeld, Y. Segev, *A uniform characterization of the octonions and the quaternions using commutators,* submitted.

NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

Color Quasi-Lie algebras and Hom-algebras

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This talk will be devoted to non-associative color Hom-algebra structures, with emphasis on color Hom-Lie Hom-algebras, color quasi Lie Hom-algebras and color Hom-Leibniz Hom-algebras [2, 4–6]. These structures extend and link in natural way color Lie algebras and Lie superalgebras with various algebras of discrete and twisted vector fields arising from q-deformed vertex operators structures and q-deferential calculus, various classes of multiparameter deformations of associative and non-associative algebras appearing in other contexts in Mathematics, Mathematical Physics, including some well-known and new one-parameter and multi-parameter deformations of infinite-dimensional Lie algebras of Witt and Virasoro type from conformal field theory, string theory and deformed vertex models, as well as multi-parameter families of quadratic and almost quadratic algebras that include for natural special "limiting" choices of parameters algebras appearing in non-commutative algebraic geometry. Common unifying feature for all these algebras is appearance of some twisted generalizations of Jacobi and Leibniz identities providing new structures of interest for investigation from the side of non-associative algebras, non-commutative and non-associative differential calculi beyond usual differential calculus, and generalized central extensions of Hom-algebra structures [1, 3]. Admissibility in context of the color Hom-algebra structures, and examples and some new results on interplay between various classes of non-associative algebras in context color Hom-algebras will be discussed. Color n-ary Hom-algebra generalizations of Nambu-Filipov algebras and related non-associative algebras will be described and some inductive constructions of n-ary hom-algebras of increasing arity, using traces on Hom-algebras, will be discussed [7–12].

Keywords: color quasi Lie Hom-algebras, Hom-Lie algebras, Hom-algebras

Mathematics Subject Classification 2020: 17A30, 17A36, 17B61

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Generalized Kantor pairs

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Quadratic Jordan algebras and pairs provide a uniform way to study algebraic groups with 3-graded Lie algebras over fields of arbitrary characteristic. Moreover, over arbitrary commutative rings one can construct from a quadratic Jordan pair or algebra a 3-graded Lie algebra with an associated "projective elementary group".

Kantor pairs, a structure similar to Jordan pairs, are useful in the study of 5-graded Lie algebras and related algebraic groups. However, over fields of low characteristic and definitely over arbitrary commutative rings, the Kantor pairs do not always induce a group.

We introduce the notion of a generalized Kantor pair defined over an arbitrary commutative ring. With such a pair we can always associate a group, generalizing the projective elementary group for Jordan pairs.

Keywords: Kantor pair, Lie algebra, Projective elementary group

Mathematics Subject Classification 2020: 17A40, 17B45, 17B70

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Poisson structure on the invariants of pairs of matrices

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The study of the ring of invariants of a set of $n \times n$ matrices is a classical problem in invariant theory. The group $\mathrm{GL}_n(\mathbb{C})$ act on the direct product $\mathcal{M}_n^d$ of the set of $n \times n$ complex matrices by simultaneous conjugation. This induces an action of GL_n on the algebra $\mathbb{C}[\mathcal{M}_n^d]$ of polynomial functions on $\mathcal{M}_n^d$ and defines the ring of invariants of $\mathbb{C}[\mathcal{M}_n^d//\mathrm{GL}_n]$. The First Fundamental Theorem of Matrix Invariants states that it is generated as a $\mathbb{C}$-algebra by the traces of monomials in the generic matrices $X_1, \ldots, X_d$. The Second Fundamental Theorem states that all relations follow from the Cayley-Hamilton theorem. The presentation of the ring of invariants of matrices in terms of generators and relations is an interesting question. While the ring of invariants $\mathbb{C}[(\mathcal{M}_2 \times \mathcal{M}_2)//\mathrm{GL}_2]$ is freely generated by $\mathrm{Tr}(X), \mathrm{Tr}(Y), \mathrm{Tr}(X^2), \mathrm{Tr}(XY), \mathrm{Tr}(Y^2)$, it becomes tedious starting with $n = 3$. Teranishi [3] found a minimal system of eleven generators of $\mathbb{C}[(M_3 \times M_3)//\mathrm{GL}_3]$ and it is not free as a $\mathbb{C}$-algebra. A huge algebraic relation was established by Nakamoto [2]. Apparently, there are no other defining relations in the coordinate ring of $(M_3 \times M_3)//\mathrm{GL}_3$ and Aslaksen, Drensky and Sadikova [1] noted that this polynomial can be considerably simplified if the generators are transformed to their "traceless" versions.

In this talk, we give an alternative way to establish this defining relation and possible generalizations. We use a completely novel approach, based on inducing a Poisson algebra structure on the ring of invariants of pairs of matrices. The Lie bracket of the Poisson algebra structure on invaraints comes from the so-called Necklace Lie algebra, observed by Kontsevich. Furthermore, we were able to further simplify the defining relation using a set of defining relations of the coordinate ring of the Calogero-Moser space and the commuting variety.

Keywords: Poisson algebra, invaraint theory

Mathematics Subject Classification 2020: 16R30, 17B63

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Homogeneous superderivations with nilpotent values in semiprime associative superalgebras

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Back in 1981, A. Giambruno and I. N. Herstein showed if R is a semiprime ring and d a derivation of R which there exists a fixed n such that $(d(x))^n = 0$ for all $x \in R$, then $d = 0$, see [1]. Many generalization of this result can be found in the literature. In this talk we give a similar result in super-setting. In particular, let $A = A_{\bar{0}} \oplus A_{\bar{1}}$ be a semiprime associative superalgebra and $d_{\bar{i}}$ be a homogeneous superderivation of degree $\bar{i} \in \mathbb{Z}_2$ of A such that $d_{\bar{i}}$ takes nilpotent values of bounded index on homogeneous elements in a two-sided graded ideal $I = I_{\bar{0}} \oplus I_{\bar{1}}$, i.e., for a fixed $\bar{i} = \bar{0}, \bar{1}$, $(d_{\bar{i}}(x_{\bar{j}}))^{n_{\bar{j}}} = 0$ for all $x_{\bar{j}} \in I_{\bar{j}}$ with $\bar{j} = \bar{0}, \bar{1}$, implies that $d_{\bar{0}}(I) = 0$ and, under some torsion assumptions depending on $n_{\bar{1}}$, $d_{\bar{1}}(I) = 0$. Moreover, for $\bar{i} = \bar{0}$, if I is a left-sided graded ideal then $Id_{\bar{0}}(I) = 0 = d_{\bar{0}}(I)d_{\bar{0}}(I)$.

Keywords: superderivation, nilpotent values, semiprime superalgebra

Mathematics Subject Classification 2020: 16N60, 16W25, 17A70

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7 Posters

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Faulkner construction for generalized Jordan superpairs

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In this poster, some definitions and results related to the Faulkner construction for generalized Jordan superpairs are given.

Keywords: Lie superalgebras, Generalized Jordan superpairs, Lie supermodules, Faulkner construction

Mathematics Subject Classification 2020: 17B60, 17C50, 17C70

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Upper triangular matrices of graded division algebras and their identities

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We consider the algebra $A = UT(D_1, \ldots, D_n)$ of upper block triangular matrices determined by an n-uple of G-graded division algebras, whose elements appear in the diagonal blocks of A. Here we consider elementary gradings on M_d which are compatible with the given gradings of $D_1, \ldots, D_n$. We prove that any two such algebras are G-isomorphic if and only if they satisfy the same graded polynomial identities. We discuss the number of different classes of isomorphic gradings and we exhibit a connection with the factorability of $Id_G(A)$. We also give some results about the generators of this T_G-ideal.

Keywords: Graded algebras, Graded polynomial identities, Upper block triangular matrix algebras

Mathematics Subject Classification 2020: 16W50, 16R10, 16R50

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Fine decompositions on algebraic systems induced by bases

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We consider algebraic systems with several products $\mathfrak{S}$ (as examples we can take algebras, superalgebras, hom-algebras, Poisson algebras, Leibniz-Poisson algebras, Bol algebras etc.) We show that any basis of $\mathfrak{S}$ gives us a decomposition of $\mathfrak{S}$ as a certain orthogonal direct sum of well-described ideals. The above decomposition result is applied to the setup of algebraic systems admitting a multiplicative basis.

Keywords: Algebraic system, Structure theory, Multiplicative basis.

Mathematics Subject Classification 2020: 17A01, 17A60, 17xx.

Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Naturally graded Lie and Leibniz superalgebras

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The study of gradations has always represented a cornerstone in the study of non-associative algebras. In particular, *natural gradation* can be considered to be the first and most relevant gradation of nilpotent Leibniz (resp. Lie) algebras. In fact, many families of relevant solvable Leibniz (resp. Lie) algebras have been obtained by extensions of naturally graded algebras, i.e. solvable algebras with a well-structured nilradical. Thus, we begin the present work introducing the concept of naturally graded for Lie and Leibniz superalgebras. Secondly we tackle the problem of determining which filiform Lie superalgebras are naturally graded obtaining a complete classification (up to isomorphism) for total dimension less or equal to 7 as well as some other classifications. Finally, we characterize naturally graded Leibniz superalgebras with maximal super-nilindex. Note that the techniques used for Leibniz superalgebras are totally different from the ones used for Lie superalgebras and involve a huge amount of computation. [1].

Keywords: Lie (super)algebras, cohomology, deformation, Leibniz (super)algebras, naturally graded

Mathematics Subject Classification 2020: 17A32, 17B30, 17B70

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Gradings on Kantor systems of Hurwitz type

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In 1972 a correspondence between Kantor pairs and 5-graded Lie algebras was introduced. A 5-graded Lie algebra can be obtained from a Kantor pair by means of a generalised Tits-Kantor-Koecher (TKK) construction. Conversely, a Kantor pair can be obtained from any 5-graded Lie algebra considering the homogeneous components of degree -1 and 1.

Our study was focused on Kantor pairs and triple systems associated to Hurwitz algebras (unital composition algebras). In this poster a classification, up to equivalence, of fine gradings by abelian groups on these Kantor pairs and triple systems over an algebraically closed field of characteristic different from 2 is given. Furthermore, the (simple) Lie algebras obtained from these Kantor pairs are shown.

Keywords: gradings, Kantor pairs, Lie algebras

Mathematics Subject Classification 2020: MSC17

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Cross products, automorphisms, and gradings

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We determine the automorphism group scheme of r-fold vector cross products. Moreover, their gradings by abelian groups are classified up to isomorphism [1].

Keywords: Cross product, Automorphism group scheme, Grading

Mathematics Subject Classification 2020: 17A30, 17A36, 15A69

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

On commutative finite-dimensional nilalgebras

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First, we will talk about a problem in algebra, in relation to determinate the additional conditions that are needed to insure the commutativity of A when A is an arbitrary algebra. We will describe some results about this problem as well as new results about algebras satisfying the identity $x^2y^2 = (xy)^2$. Finally we will give new approaches to a classical problem proposed by Albert [1] concerning the existence of simple commutative finite-dimensional (power-associative) nilalgebras and some properties about the modules over commutative power-associative nil algebras of low dimension [2]. Unless explicitly stated otherwise therein, all the algebras are over a field F with characteristic different from 2 and 3.

Keywords: power-associative algebras, Albert's Problem

Mathematics Subject Classification 2020: 17A05, 17A60

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

An approach to the classification of finite semifields by quantum computing

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Finite semifields are finite nonassociative rings with an identity element, such that the set of nonzero elements is a loop under the product. They were considered first by Dickson [2], and studied by Albert [1] and Knuth [3]. Finite Semifields of order 16 have been classified by Kleinfeld in [7], of order 32 by Knuth in [3] and by Walker in [9]. The classification of finite semifields can be rephrased as a problem of finding certain sets of matrices, which can be solved by computer search. So, classification of semifields with high order such as 64, was achieved by Rúa, Combarro, Ranilla in [4], or of order 243 by Rúa, Combarro, Ranilla in [5]. Based on this approach, classification of finite semifields of any order via a quantum procedure is possible. We present quantum techniques for the classification of semifields with 8 and 16 elements with their respective simulations, based on Grover's quantum search algorithm.

Keywords: Finite Semifields, Grover Algorithm, Classification

Mathematics Subject Classification 2020: 12k10, 17-04, 17-08

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NON-ASSOCIATIVE ALGEBRAS AND RELATED TOPICS II
IN HONOUR OF ALBERTO ELDUQUE ON THE OCCASION OF HIS 60TH BIRTHDAY,
Coimbra, July 18–22, 2022

A Jordan canonical form for nilpotent elements in an arbitrary ring

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In this work we give a Jordan canonical form for a nilpotent element a in a ring R such that a^k is von Neumann regular for every $k \in \mathbb{N}$ and extend it to algebraic elements in associative regular algebras over algebraically closed fields. This result makes it possible to characterize prime rings of bounded index n with a nilpotent element $a \in R$ of index n and von Neumann regular a^{n-1} as a matrix ring over a unital domain..

Keywords: Jordan canonical form, arbitrary ring

Mathematics Subject Classification 2020: 16E50, 16U50

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

Homogeneous spaces of G_2

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Pilar Benito, Cristina Draper and Alberto Elduque study the reductive homogeneous spaces obtained as quotients of the exceptional group G_2 in the Draper doctoral dissertation (see [1]), from an algebraic perspective. In this poster we revisit these spaces from a more geometrical approach.

Keywords: G_2, homogeneous spaces, exceptional Lie groups

Mathematics Subject Classification 2020: 53C30, 17B25

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Non-Associative Algebras and Related Topics II
in honour of Alberto Elduque on the occasion of his 60th birthday,
Coimbra, July 18–22, 2022

On isomorphism conditions for algebra functors with applications to Leavitt path algebras

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We introduce certain functors from the category of commutative rings (and related categories) to that of $\mathbb{Z}$-algebras (not necessarily associative or commutative). One of the motivating examples is the Leavitt path algebra functor $R \mapsto L_R(E)$ for a given graph E. Our goal is to find "descending" isomorphism results of the type: if $\mathcal{F}, \mathcal{G}$ are algebra functors and $K \subset K'$ a field extension, under what conditions an isomorphism $\mathcal{F}(K') \cong \mathcal{G}(K')$ of K'-algebras implies the existence of an isomorphism $\mathcal{F}(K) \cong \mathcal{G}(K)$ of K-algebras? We find some positive answers to that problem for the so-called "extension invariant functors" which include the functors associated to Leavitt path algebras, Steinberg algebras, path algebras, group algebras, evolution algebras and others. For our purposes, we employ an extension of the Hilbert's Nullstellensatz Theorem for polynomials in possibly infinitely many variables, as one of our main tools. We also remark that for extension invariant functors $\mathcal{F}, \mathcal{G}$, an isomorphism $\mathcal{F}(H) \cong \mathcal{G}(H)$, for some Hopf K-algebra H, implies the existence of an isomorphism $\mathcal{F}(S) \cong \mathcal{G}(S)$ for any commutative and unital K-algebra S.

Keywords: Isomorphism of K-algebras, functor, prime field, extension of fields, Cohn and Leavitt path algebras.

Mathematics Subject Classification 2020: 16S88, 16S99, 18A05, 18A22