ALGEBRAS WHOSE RIGHT NUCLEUS IS THE EIGENRING OF A SKEW POLYNOMIAL

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Let D be a division ring. We define a class of unital nonassociative algebras, which can be obtained via a construction involving the skew polynomial ring $R = D[t;\sigma]$. Such algebras were introduced by Petit in 1966 and are canonical generalizations of the associative quotient algebras R/Rf for some two-sided polynomial $f(t) \in R$. The right nucleus of these so-called Petit algebras is equal to the eigenring of the skew polynomial $f \in R$. In the literature, this eigenring is commonly used to determine the decomposability or irreducibility of a skew polynomial, however, it has never been studied in this nonassociative context before. This talk is based on joint work with my PhD supervisor Susanne Pumplün.

GRADED-SIMPLE ALGEBRAS AND LOOP ALGEBRAS

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The loop algebra construction by Allison, Berman, Faulkner, and Pianzola, describes graded-central-simple algebras with split centroid in terms of central simple algebras graded by a quotient of the original grading group. This construction will be reviewed, and the restriction on the centroid will be removed, at the expense of allowing some deformations (cocycle twists) of the loop algebras.

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PRETORSION THEORIES IN ARBITRARY CATEGORIES

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I will present two examples of pretorsion theories $(\mathcal{T}, \mathcal{F})$ on a category $\mathcal{C}$. The first example appears in [1]. In this case, the category $\mathcal{C}$ is the category of all non-empty preordered sets, $\mathcal{T}$ is the class of all non-empty sets with an equivalence relation, and $\mathcal{F}$ is the class of all non-empty sets with a partial order. The second example appears in [3]. Here now the category $\mathcal{C}$ is the category of all mappings $f \colon X \to X$, with X any finite non-empty set, $\mathcal{T}$ is the class of all bijections $f \colon X \to X$, and $\mathcal{F}$ is the class of all mappings $f \colon X \to X$ whose associated directed graph is a forest. This leads us to define a notion of pretorsion theory [2]. Torsion theories have been studied for the past twenty years for several particular types of non-additive categories.

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A CHARACTERIZATION OF THE SEMISIMPLICITY OF LIE-TYPE ALGEBRAS THROUGH THE EXISTENCE OF CERTAIN LINEAR BASES

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The class of *Lie-type algebras* contains the ones of associative algebras, Lie algebras and Leibniz algebras among other classes of algebras. We show that a Lie-type algebra $\mathcal{A}$, of arbitrary dimension and over an arbitrary base field, is semisimple if and only if it has zero annihilator and admits a weak-division linear basis. As a corollary, the simplicity of $\mathcal{A}$ is also characterized.

SUPER JORDAN TRIPLE SYSTEMS

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In this talk we will introduce the *supersymmetric Jordan triple systems*: a new algebraic structure which generalizes the class of Jordan triple systems [4] as well as the class of N=6 3-algebras [1]. We will describe their relation with graded Lie superalgebras with involutions via the Tits-Kantor-Koecher construction. Their classification, obtained via the TKK construction using the methods developed in [2], will be discussed and explicit examples of the systems related to the special and to the exceptional Lie superalgebras will be given. We will show that these systems are related to $(-1,-1)$ balanced Freudenthal-Kantor triple systems classified by Elduque et al. in [3].

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ASSOCIATIVE-ADMISSIBLE ALGEBRAS

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An algebra A is called *Lie-admissible* (LiA), if its minus algebra $A^{(-)} = (A, [\ ,])$ is Lie, *associative-admissible* (AsA) if its plus-algebra $A^{(+)} = (A, \{\ ,\})$ is associative, $< a, b, c >= 0$, for any $a, b, c \in A$, where $[a, b] = ab - ba, \{a, b\} = ab + ba$ and $< a, b, c >= \{a, \{b, c\}\} - \{\{a, b\}, c\}$. Non-commutative Lie algebra (NCL) is an algebra that satisfies Jacobi identity and all consequences of skew-symmetric identity in degree 3. Let us introduce some classes of algebras.

Name of algebras	identities
Reverse-associative	revas=$t_1(t_2t_3) - (t_3t_2)t_1$
Anti-reverse-associative	arevas=$t_1(t_2t_3) + (t_3t_2)t_1$
Left-weak-Leibniz	lwlei=$[t_1, t_2]t_3 - 2t_1(t_2t_3) + 2t_2(t_1t_3)$
Right-weak-Leibniz	rwlei=$t_1[t_2, t_3] - 2(t_1t_2)t_3 + 2(t_1t_3)t_2$
Weak-Leibniz	lwlei, rwlei
Non-commutative Lie =Two-sided Leibniz	$(t_1t_2)t_3 + (t_2t_3)t_1 + (t_3t_1)t_2$, $(t_1t_2 + t_2t_1)t_3$, revas

Theorem 1. *Associative-admissible operad is Koszul and its dual is non-commutative Lie operad, $AsA^! = NCL$. Dimensions of multilinear parts of AsA in degree n is equal to*

$$\left(\frac{1+x}{1-x-x^2}\ \frac{d}{dx}\right)^{n-1}\left(\frac{1+x}{1-x-x^2}\right)\Bigg|_{x->0}.$$

Let $AsLiA = AsA \cap LieA$ be associative- and Lie-admissible operad. Then $AsLiA$ is Koszul. Its Koszul dual is reverse-associative and (one-sided) weak-Leibniz operad, $AsLiA^! = RevAs \cap LwLei$. Dimensions of multi-linear parts of $AsLiA$ in degree n is equal to

$$\left(\frac{e^x(1+x)}{1+(1-e^x)x}\ \frac{d}{dx}\right)^{n-1}\left(\frac{e^x(1+x)}{1+(1-e^x)x}\right)\Bigg|_{x->0}.$$

Example. Let $U = K[x]$ and $a \star b = a\partial(b) - \partial(a)b + uab$, for some $u \in U$. Then $(U, \star)$ is weak-Leibniz. It is associative-admissible and Lie-admissible.

Well known that if Leibniz algebra is simple, then it is Lie. Any Leibniz algebra is weak-Leibniz. Our example shows that the class of weak Leibniz algebras contains simple algebras, that are not Leibniz and not Lie.

Theorem 2. *Weak Leibniz operad is self-dual, $WLei^! = WLei$, but it is not Koszul.*

Theorem 3. *Let X be set of generators, $F(X)$ free reverse-associative algebra, $Com(X)$ free commutative algebra and $ACom(X)$ free anti-commutative algebra. Then $F(X) \cong Com(X) \oplus ACom(X)$. In particular, dimensions of multi-linear parts of free reverse-associative algebra in degree $n > 1$ is $2(2n-3)!!$. Koszul dual of reverse-associative operad is anti-reverse-associative operad, $RevAs^! = ARevAs$.*

ALMOST INNER DERIVATIONS OF LIE ALGEBRAS

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An interesting question in spectral geometry posed by Hermann Weyl was whether or not isospectral manifolds are necessarily isometric. That this is not always the case, Milnor showed in 1964 by giving two isospectral non-isometric flat tori in 16 dimensions. In 1984 however, Gordon and Wilson were the first to construct continuous families of isospectral non-isometric manifolds instead of only finite ones. Therefore, they introduced the notion of almost inner derivations of Lie algebras [2].

A derivation D of a Lie algebra $\mathfrak{g}$ is called almost inner if and only if it is a derivation where every element is adjoint to its image. This means that for every $x \in \mathfrak{g}$, there exists $y \in \mathfrak{g}$ such that $D(x) = [x, y]$ (where y can depend on x). Hence, this condition is less strict than the one for an inner derivation, where every element is mapped to the Lie bracket of itself with some fixed element.

The geometric motivation to study almost inner derivations only makes sense for nilpotent Lie algebras over the real and complex field. However, from an algebraic point of view, there is no reason to restrict to these Lie algebras and these fields. In [1], the first algebraic results on this topic were obtained. The talk will consist of definitions, some examples and the main results concerning almost inner derivations of Lie algebras.

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NONASSOCIATIVE STRUCTURES AND 3-SASAKIAN HOMOGENEOUS MANIFOLDS

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The 3-Sasakian homogeneous spaces are certain contact manifolds whose geometric structure is very well codified in Lie theoretical terms. This fact can be used to find interesting invariant affine connections, with nice properties or special holonomies. The more fruitful results arise in the particular case of the 7-dimensional 3-Sasakian homogeneous manifolds, that is, the corresponding sphere and the Aloff-Wallach space, although the target is to find a good connection independently of the dimension.

A nonassociative structure related to Lie algebras appears through this study, that one of symplectic triple system. In particular the curvature of an affine connection can be written by means of the binary and ternary products in this triple system. Also, for some distinguished connections, the holonomy algebra can be described in a unified form, what helps to find a suitable candidate for a *best affine connection adapted to the geometry* of the 3-Sasakian manifold, which necessarily will have nonzero torsion.

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POST-LIE ALGEBRA STRUCTURES ON CLASSES OF NILPOTENT LIE ALGEBRAS

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Post-Lie algebra structures arise in many different contexts, e.g., in connection with homology of partition sets, operad theory, geometric structures on Lie groups, crystallographic groups, Yang-Baxter equations, Rota-Baxter operators and other topics.

We will present results on (commutative) post-Lie algebra structures on some classes on nilpotent Lie algebras. For non-metabelian filiform nilpotent Lie algebras and Lie algebras of strictly upper triangular matrices we show that all commutative post-Lie algebra structures are associative and induce an associated Poisson-admissible algebra. As a corollary, we obtain a classification of all commutative post-Lie algebra structures on certain important classes of filiform Lie algebras.

This is joint work with D. Burde.

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A GENERALISATION OF DICKSON'S COMMUTATIVE DIVISION ALGEBRAS

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Dickson's commutative semifields are an important class of finite division algebras. We generalise Dickson's construction by doubling both finite field extensions and central simple algebras and not restricting us to the classical setup where a cyclic field extension is taken. The latter case yields algebras which are no longer commutative nor associative. It turns out that the conditions for when the algebras are division algebras canonically generalise the classical ones. We compute all the automorphisms, including the structure of the automorphism group in some cases.

LEIBNIZ A-ALGEBRAS

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A finite-dimensional Lie algebra is called an A-algebra if all of its nilpotent subalgebras are abelian. These arise in the study of constant Yang-Mills potentials and have also been particularly important in relation to the problem of describing residually finite varieties. They have been studied by several authors, including Bakhturin, Dallmer, Drensky, Sheina, Premet, Semenov, Towers and Varea. In this talk I will give generalisations of many of these results to Leibniz algebras.

CODIMENSION GROWTH AND MINIMAL VARIETIES

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In characteristic zero an effective way of measuring the polynomial identities satisfied by an algebra is provided by the sequence of its codimensions introduced by Regev. In this talk we review some features of the codimension growth of PI algebras, including the deep contribution of Giambruno and Zaicev on the existence of the PI-exponent, and discuss some recent developments in the framework of group graded algebras. In particular, a characterisation of minimal supervarieties of fixed superexponent will be given. The last result is part of a joint work with O.M. Di Vincenzo and V.R.T. da Silva.

ON LEIBNIZ COHOMOLOGY

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This is joint work together with Jörg Feldvoss (University of South Alabama, Mobile, USA). We prove in this work the Leibniz analogues of several vanishing theorems for the Chevalley-Eilenberg cohomology of Lie algebras. In particular, we obtain the second Whitehead lemma for Leibniz algebras. Our main tools are three spectral sequences. Two are Leibniz analogues of the Hochschild-Serre spectral sequence, one of which is dual to a spectral sequence of Pirashvili [4] for Leibniz homology and the other one is due to Beaudouin [1]. A third spectral sequence relates the Leibniz cohomology of a Lie algebra to its Chevalley-Eilenberg cohomology [4]. The vanishing theorems are illustrated with several computational examples both in characteristic zero and in positive charcateristic.

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A WAY OF CONSTRUCTION OF LEIBNIZ ALGEBRAS VIA REPRESENTATIONS OF LIE ALGEBRAS

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The work is devoted to finding minimal linear representations of some classes of filiform Lie algebras of dimension n. Specifically, we find minimal faithful representations of the filiform Lie algebras Q_{2n}, R_n and W_n. Moreover, we construct Leibniz algebras with corresponding Lie algebras Q_6, R_7 and W_5, by using these representations of filiform Lie algebras.

ON LESIEUR-CROISOT ELEMENTS OF JORDAN PAIRS

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A Jordan algebra J is a Lesieur-Croisot algebra if J is a classical order in a nondegenerate unital Jordan algebra having finite capacity. Lesieur-Croisot Jordan algebras were characterized by Fernández López, García Rus and Montaner as those nondegenerate Jordan algebras satisfying the property that an inner ideal is essential if and only if it contains an injective element.

Later Montaner and Tocón studied the set of Lesieur-Croisot elements of a nondegenerate Jordan algebra J, that is the set of all elements of the nondegenerate Jordan algebra J such that their local algebras are Lesieur-Croisot algebras. This set, denoted $LC(J)$, played a fundamental role for the development of the local Goldie theory for Jordan algebras recently given by Montaner and Paniello.

In this talk, aimed to develop a general localization theory for Jordan pairs, we will consider the set $LC(V)$ of Lesieur-Croisot elements of a nondegenerate Jordan pair V and characterize nondegenerate Jordan pairs satisfying $V = LC(V)$.

This a joint work with Fernando Montaner (University of Zaragoza, Spain).

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STRUCTURABLE ALGEBRAS, TITS-KANTOR-KOECHER LIE ALGEBRAS AND THEIR INNER IDEALS

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Structurable algebras are a class of non-associative algebras introduced by Allison in [1] which includes the class of Jordan algebras. In [1] Allison also introduced a procedure to associate a 5-graded Lie algebra $K(\mathcal{A})$ to any structurable algebra $\mathcal{A}$, generalizing the Tits-Kantor-Koecher construction for Jordan algebras. The other main ingredient for this talk will be the concept of an inner ideal. An inner ideal of a Lie algebra L is a subspace I such that $[I, [I, L]] \leq I$. There exists a notion of inner ideals for structurable algebras as well.

For each structurable division algebra we describe the inner ideals of its associated Lie algebra and relate these to so-called Moufang sets. We also describe the inner ideals of the Lie algebras associated to a specific class of skew-dimension one structurable algebras and we give these inner ideals the structure of a generalized hexagon. There are some connections with the extremal geometries defined by Cohen and Ivanyos in [2]. This is joint work with Tom De Medts.

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NONASSOCIATIVE PRODUCTS IN NEARLY ASSOCIATIVE ALGEBRAS WITH INVOLUTION

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Starting from the variety of associative, Lie, Jordan, alternative or Malcev algebras with involution (resp. with involutive automorphism), we classify all the formal bilinear products of the form

$$axy + bx^*y + cxy^* + dx^*y^* + Ayx + Byx^* + Cy^*x + Dy^*x^*$$

formed with the help of the original product and the involution $*$ (resp. involutive automorphism), which are either flexible, power-associative, alternative, associative, Jordan, binary-Lie, Malcev or Lie for all algebras of the chosen variety. To do so we formally define and study the change of product in an algebra acted on by a group of automorphisms and antiautomorphisms, solve the associative variety case via free algebras with computer assistance, and then solve the nonassociative cases via representations, nonassociative PI theory, and specific algebras.

AXIAL ALGEBRAS

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We will give an introduction to axial algebras and discuss some recent developments. Axial algebras are a new class of non-associative algebra, introduced recently by Hall, Rehren and Shpectorov, which generalise some properties found in vertex operator algebras and the Griess algebra. Axial algebras are generated by *axes* which are idempotents which decompose the algebra as a direct sum of eigenspaces. The multiplication of eigenvectors is controlled by a so-called *fusion law*. When this is graded, it leads naturally to a subgroup of automorphisms of the algebra called the *Miyamoto group*. The prototypical example is the Griess algebra which has the Monster simple sporadic group as its Miyamoto group. Other examples include a wide number of Jordan algebras and also Matsuo algebras, which are defined from 3-transposition groups.

COMMUTATIVE POST-LIE ALGEBRA STRUCTURES AND LINEAR EQUATIONS FOR NILPOTENT LIE ALGEBRAS

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In this talk we will explain the results we obtained in [1]. In that paper, we study commutative post-Lie algebra structures, or CPA-structures. A CPA structure on a Lie algebra $\mathfrak{g}$ with Lie bracket $[\cdot,\cdot]$ is a bilinear product on $\mathfrak{g}$ satisfying

$$x \cdot y = y \cdot x$$
$$[x,y] \cdot z = x \cdot (y \cdot z) - y \cdot (x \cdot z)$$
$$x \cdot [y,z] = [x \cdot y, z] + [y, x \cdot z]$$

After recalling the motivation to study these CPA structures, we will show that for a given nilpotent Lie algebra $\mathfrak{g}$ with $Z(\mathfrak{g}) \subseteq [\mathfrak{g},\mathfrak{g}]$ all CPA-structures on $\mathfrak{g}$ are complete. This means that all left and all right multiplication operators in the algebra are nilpotent. Then we study CPA-structures on free-nilpotent Lie algebras $F_{g,c}$ and discover a strong relationship to solving systems of linear equations of type $[x,u]+[y,v]=0$ for generator pairs $x, y \in F_{g,c}$. We use results of Remeslennikov and Stöhr [2] concerning these equations to prove that, for certain g and c, the free-nilpotent Lie algebra $F_{g,c}$ has only central CPA-structures.

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NILPOTENT TORTKARA ALGEBRAS OF DIMENSION 6

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An anticommutative algebra **A** is called a *Tortkara algebra* if it satisfies the identity

$$(ab)(cb) = J(a, b, c)b, \text{ where } J(a, b, c) = (ab)c + (bc)a + (ca)b.$$

Tortkara algebras were introduced by A. Dzhumadildaev in [1]. For example, any Zinbiel algebra with the commutator multiplication is a Tortkara algebra.

We classify all 6-dimensional nilpotent Tortkara algebras as central extensions [2] of algebras of dimension less than 6. We obtain a list of 20 pairwise non-isomorphic nilpotent Tortkara non-Malcev algebras, of which only one algebra is non-metabelian.

This is a joint work with Ilya Gorshkov (Sobolev Institute of Mathematics, Russia) and Ivan Kaygorodov (Universidade Federal do ABC, Brazil) [3].

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ON SOME PROPERTIES PRESERVERD BY THE NON-ABELIAN TENSOR PRODUCT OF *HOM*-LIE ALGEBRAS

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The main goal of this talk is to present a study about the preservation of some properties from Hom-Lie algebras to the non-abelian tensor product of Hom-Lie algebras.

To do so we organize the talk as follows: first we recall from [**?**] basic properties of the non-abelian tensor product of Hom-Lie algebras. As a main result we establish the conditions under which the following isomorphism holds,

$$(A, \alpha_A) \star ((B, \alpha_B) \times (C, \alpha_C)) \cong ((A, \alpha_A) \star (B, \alpha_B)) \times ((A, \alpha_A) \star (C, \alpha_C)) \, .$$

Following we prove that the non-abelian tensor product $(M \star N, \alpha_{M \star N})$ inherits the property of nilpotent (respectively, solvable or k-Engel) Hom-Lie algebra if such a property, respectively, is satisfied by one of the Hom-ideals $({}^N M =< \{{}^n m, n \in N, m \in M\} >, \alpha_{{}_{N_M}})$ (respectively $({}^M N < \{{}^m n, n \in N, m \in M\} >, \alpha_{{}_{M_N}}))$ of (M, α_M) and (N, α_N). We also introduce the concept of nilpotent Hom-action and we establish a relationship between this property over the non-abelian tensor product and the nilpotency of a quotient Hom-Lie algebra.

Finally, we analyze the conditions under which the non-abelian tensor product of Hom-Lie algebras preserves perfect objects. As main result we establish the relationship between the universal central extension of the non-abelian tensor product of two perfect Hom-Lie algebras and the non-abelian tensor product of the universal central extensions corresponding to every perfect Hom-Lie algebra.

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EMBEDDABLE ALGEBRAS INTO ZINBIEL ALGEBRAS VIA THE COMMUTATOR

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An algebra with identity $a(bc) = (ab)c + (ba)c$ is called *(right)-Zinbiel*. These algebras were introduced by J-L.Loday in [1] as dual of Leibniz algebras.

If A is a Zinbiel algebra, then the plus algebra $A^{(+)} = (A, +, \{,\})$ is commutative and associative [1], where $\{a, b\} = ab + ba$ for $a, b \in A$. The minus algebra $A^{(-)} = (A, +, [,])$ satisfies non-trivial identity so-called *Tortkara* identity [2]

$$[[a, b], [c, b]] = [J[a, b, c], b]$$

where $J[a, b, c] = [[a, b], c]+[[b, c], a]+[[c, a], b]$ and $[a, b] = ab-ba$ for $a, b \in A$. We give Lie and Jordan criterions for free Zinbiel algebras. We consider examples of special and exceptional Tortkara algebras. Further, we prove analogue of classical Cohn's and Shirshov's theorems for Tortkara algebras.

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COMMUTATIVE COHOMOLOGY IN CHARACTERISTIC 2

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Commutative Lie algebras are commutative algebras in characteristic 2 satisfying the Jacobi identity. This class of algebras is broader than the class of Lie algebras (which satisfy the stronger alternating identity [x,x] = 0). The natural cohomology in this class is a variation of the usual Chevalley cohomology, where alternating cochains are replaced by symmetric ones. This cohomology appears naturally in the ongoing efforts to advance in classification of simple Lie algebras in characteristic 2, and its properties resemble cohomology of Lie superalgebras (in any characteristic) rather than the usual Chevalley cohomology. We will discuss some general results, computations, and open questions.

HOM-ASSOCIATIVELY DEFORMED WEYL ALGEBRAS

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In this talk, I will give an introduction to so-called hom-associative deformations of the classically rigid Weyl algebras, as well as some other examples. Moreover will I describe the general framework in which they are defined: the non-associative, non-commutative polynomial rings known as hom-associative Ore extensions. If time permits, I will show how a Hilbert's basis theorem can be generalized to this particular, and a general non-associative, setting.

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ON WHITEHEAD'S QUADRATIC FUNCTOR FOR SUPERMODULES

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Simson and Tyc gave a generalized version of the Whitehead's quadratic functor $\Gamma(M)$ for a module M, following the traditional presentation as the free algebra generated by M, subject to certain relations. It determines a quadratic mapping $\gamma\colon M \to \Gamma(M)$, which is universal in the sense that for any other quadratic mapping $f\colon M \to N$, there exists a unique homomorphism of modules $h\colon \Gamma(M) \to N$ such that $h\gamma = f$.

In our work, we generalize this object for supermodules, following an alternative construction much more suitable for working with quadratic mappings between supermodules. We explore some of its fundamental properties. Also, we generalize it to abelian crossed modules of Lie superalgebras, and we use this new construction to define quadratic mappings between abelian crossed modules of Lie superalgebras.

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DERIVED LENGTHS OF SYMMETRIC POISSON ALGEBRAS

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Let L be a Lie algebra over a field of characteristic $p > 0$ and let $S(L)$ and $\mathbf{s}(L)$ denote, respectively, the symmetric Poisson algebra and the truncated symmetric Poisson algebra of L. We investigate the structure of L when $S(L)$ or $\mathbf{s}(L)$ is solvable. A conjecture of Monteiro Alves and Petrogradsky about solvability of $S(L)$ in characteristic 2 is disproved. Moreover, the derived lengths of $\mathbf{s}(L)$ are studied. In particular, we provide bounds for the derived lenghts of $\mathbf{s}(L)$, establish when $\mathbf{s}(L)$ is metabelian, and characterize truncated symmetric Poisson algebras of minimal derived length.

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REPRESENTATION THEORY OF MAP SUPERALGEBRAS

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Representation theory of map (super)algebras has been a topic which has seen a lot of development in the last tweenty years, due to its close relations to the theory of Kac-Moody algebras. A map superalgebra is a Lie superalgebra constructed in the following way. Let k be an algebraically closed field of characteristic zero, $\mathfrak{g}$ be a finite-dimensional simple Lie superalgebra over k, and A be an associative commutative finitely-generated unital algebra over k. A map superalgebra is obtained as the tensor product $\mathfrak{g} \otimes_k A$. In this talk we will present several results related to the classification of finite-dimensional modules for map superalgebras and the study of their extensions.

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ALGORITHMIC METHODS FOR LIE ALGEBRAS

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I will give a *survey* on algorithmic and experimental methods for Lie algebras and their current implementation in the computer algebra system GAP. In particular, the differences between fields of characteristic 0 and finite fields are highlighted. For the former Willem de Graaf [1] has exhibited algorithms for various problems.

For finite fields algorithms for testing isomorphism and calculating automorphism groups of finite-dimensional Lie algebras are available: Schneider [4] has developed a method for the nilpotent case and Eick has implemented algorithms for the solvable case [2]. Eick also gives an algorithm for arbitrary finite-dimensional Lie algebras over finite fields [1]. This approach depends on "nice" presentations for the Lie algebras considered.

Finally, I will report on random search methods for simple Lie algebras over finite fields and the computation of gradings of such algebras.

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STRUCTURABLE ALGEBRAS, LIE ALGEBRAS AND ALGEBRAIC GROUPS

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Structurable algebras form a class of non-associative algebras with involution, introduced by Bruce Allison in 1978, simultaneously generalizing associative algebras with involution and Jordan algebras [1]. They have been studied very extensively by Bruce Allison, John Faulkner, Richard Schafer, Oleg Smirnov and many others. Nevertheless, many results are spread out in the literature and are not always very well known to outsiders.

Structurable algebras were introduced because of their connection with (simple) Lie algebras through the Tits–Kantor–Koecher construction. Not completely surprisingly, this connection continues to hold on the level of linear algebraic groups, but the details are sometimes delicate. For the case of structurable division algebras, this has recently been carried out in full detail in [2].

The goal of the talk is to present the theory along with many examples. We will give an overview of the connections with Lie algebras and algebraic groups. We will also include some other recent developments.

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THE CLASSIFICATION OF NILPOTENT BICOMMUTATIVE ALGEBRAS

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The variety of bicommutative algebras is defined by the following identities of right- and left-commutativity:

$$(xy)z \;=\; (xz)y, \qquad x(yz) \;=\; y(xz).$$

It admits the commutative associative algebras as a subvariety. Bicommutative algebras were studied in [1–4].

The key step in our method for algebraically classifying bicommutative nilpotent algebras is the calculation of central extensions of smaller algebras. Firstly, Skjelbred and Sund devised a method for classifying nilpotent Lie algebras employing central extensions. We use this method for classifying all the 4-dimensional nilpotent bicommutative algebras.

Also we establish geometrical classification and prove that the variety of 4-dimensional nilpotent bicommutative algebras has two irreducible components defined by the one rigid algebra and the one infinite family of algebras.

This is a joint work with Pilar Páez-Guillán (Universidad de Santiago de Compostela) and Ivan Kaygorodov (Universidade Federal do ABC, Brazil) [5].

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2-GENERATION OF SIMPLE LIE ALGEBRAS AND FREE DENSE SUBGROUPS OF ALGEBRAIC GROUPS

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It is known (at least since the 50's) that a semisimple complex Lie algebra contains two elements that generate it [1]. Here we consider the problem of finding pairs of nilpotent elements that generate a given simple Lie algebra. We show several constructions (of varying generality) of such pairs for simple Lie algebras. We use these to find free Zariski dense subgroups of simple algebraic groups. Firstly, if x, y are nilpotent elements generating the Lie algebra of such a group, then $\exp(\mathrm{ad}x)$, $\exp(\mathrm{ad}y)$ generate a Zariski dense subgroup. Secondly we use the so-called ping-pong lemma to find some infinite series of simple algebraic groups where these dense subgroups are free, generalising a well known construction for $\mathrm{SL}(2, \mathbf{C})$ (see [2], Proposition 12.3 of Chapter III). This is joint work with Alla Detinko.

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CHARACTERISING ALGEBRAIC STRUCTURES VIA CATEGORICAL PROPERTIES

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By a categorical characterisation of an algebraic structure we mean a characterisation that only uses universal properties of objects and morphisms, never handling particular elements of the underlying set. In this talk we will cover briefly both solutions to the 15-year standing problem of characterising groups amongst monoids [1, 5], and its generalisation to cocommutative Hopf algebras amongst cocommutative bialgebras [2]. Then, we will talk about our new characterisation of Lie algebras amongst all non-associative algebras via algebraic exponentiation [3, 4]. We prove that Lie algebras is the only variety of non-associative algebras which is *locally algebraically cartesian closed*.

Joint work with Tim Van der Linden.

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CONSERVATIVE ALGEBRAS AND SUPERALGEBRAS

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In 1972, Kantor introduced the notion a conservative algebra as a generalization of Jordan algebras. Unlike other classes of non-associative algebras, this class is not defined by a set of identities. The class of conservative algebras is very vast. Apart from Jordan algebras, it includes all associative algebras, all Lie algebras, all (left) Leibniz algebras, and many other classes of algebras. Also it has a natural version of TKK construction and has connections to other generalizations of Jordan algebras, such as structurable algebras. In this talk we discuss classical and new results related to conservative algebras, in particular, our construction of a universal conservative superalgebra.

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