Nilpotent Tortkara algebras of dimension 6

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Contents

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Introduction

Tortkara algebras

- **A** an algebra over a field.

Definition 1.1 (Dzhumadildaev [2])

The algebra **A** is called a Tortkara algebra if it satisfies

1. $ab = -ba$;
2. $(ab)(cb) = J(a, b, c)b$,

where $J(a, b, c) = (ab)c + (bc)a + (ca)b$.

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Examples

- metabelian Lie algebras (i.e., $[[x, y], [z, t]] = 0$);
- (right-)Zinbiel algebras (i.e., $x(yz) = (xy + yx)z$) under the commutator product $[x, y] = xy - yx$.

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Our goal

Problem

Classify all 6-dimensional nilpotent Tortkara non-Malcev algebras over $\mathbb{C}$.

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Nilpotent Tortkara algebras of small dimension

Nilpotent Tortkara algebras of dimension ≤ 4

- Nilpotent algebras of dimension 1 are trivial.
- There is only one non-trivial nilpotent algebra of dimension 2: $e_1^2 = e_2$. **It is not Tortkara**.
- There is only one non-trivial nilpotent anticommutative algebra of dimension 3, and **it is Tortkara**:

$$\mathbb{T}_{01}^3 : \ e_1 e_2 = e_3.$$

- There is only one non-trivial nilpotent anticommutative algebra of dimension 4, and **it is Tortkara**:

$$\mathbb{T}_{02}^4 : \ e_1 e_2 = e_3, \ e_1 e_3 = e_4.$$

The last two items follow from [1].

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The method

Cocycles

- **A** a Tortkara algebra over $\mathbb{C}$;
- $\mathbb{V}$ a $\mathbb{C}$-vector space.

Definition 3.1

A *Tortkara (two-)cocycle* on **A** with values in V is a skew-symmetric bilinear map $\theta : \mathbf{A} \times \mathbf{A} \to \mathbb{V}$, such that

$$\theta(xy, zt) + \theta(xt, zy) = \theta(J(x, y, z), t) + \theta(J(x, t, z), y).$$

The Tortkara cocycles on **A** with values in $\mathbb{V}$ form a $\mathbb{C}$-linear space denoted by $Z_T^2(\mathbf{A}, \mathbb{V})$.

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Coboundaries

- $f \in \mathrm{Hom}(\mathbf{A}, \mathbb{V})$.

Definition 3.2

Define $\delta f : \mathbf{A} \times \mathbf{A} \to \mathbb{V}$ by

$$\delta f(x, y) = f(xy).$$

The maps $\mathbf{A} \times \mathbf{A} \to \mathbb{V}$ of the form δf are called *(two-)coboundaries* on $\mathbf{A}$ with values in $\mathbb{V}$. They form a linear subspace of $Z_T^2(\mathbf{A}, \mathbb{V})$ denoted by $B^2(\mathbf{A}, \mathbb{V})$.

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Definition 3.3

We define the *Tortkara (second) cohomology space* $H^2_T(\mathbf{A}, \mathbb{V})$ on $\mathbf{A}$ with values in $\mathbb{V}$ as the quotient space $Z^2_T(\mathbf{A}, \mathbb{V}) \big/ B^2(\mathbf{A}, \mathbb{V})$.

The algebra $\mathbf{A}_\theta$

- $\mathbf{A}$ a Tortkara algebra of dimension $m < n$;
- $\mathbb{V}$ a vector space of dimension $n - m$ over the same field;
- θ a skew-symmetric bilinear form on $\mathbf{A}$ with values in $\mathbb{V}$.

Definition 3.4

Define $\mathbf{A}_\theta := \mathbf{A} \oplus \mathbb{V}$ with the multiplication $[-,-]_{\mathbf{A}_\theta}$ given by

$$[x + x', y + y']_{\mathbf{A}_\theta} = xy + \theta(x, y),$$

where $x, y \in \mathbf{A}$ and $x', y' \in \mathbb{V}$.

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Central extensions

Proposition 3.5

$\mathbf{A}_\theta$ *is a Tortkara algebra if and only if* $\theta \in Z_T^2(\mathbf{A}, \mathbb{V})$.

Definition 3.6

If $\theta \in Z_T^2(\mathbf{A}, \mathbb{V})$, then $\mathbf{A}_\theta$ is called an $(n-m)$*-dimensional central extension* [5] of $\mathbf{A}$ by $\mathbb{V}$.

The annihilators

- $\mathbf{A}$ a Tortkara algebra;
- $\theta \in Z_T^2(\mathbf{A}, \mathbb{V})$.

Definition 3.7

The *annihilator* of $\mathbf{A}$ is the ideal $\operatorname{Ann} \mathbf{A} = \{x \in \mathbf{A} : x\mathbf{A} = 0\}$.
The *annihilator* of θ is the subspace $\operatorname{Ann} \theta = \{x \in \mathbf{A} : \theta(x, \mathbf{A}) = 0\}$.

Proposition 3.8

One has $\operatorname{Ann} \mathbf{A}_\theta = (\operatorname{Ann} \mathbf{A} \cap \operatorname{Ann} \theta) \oplus \mathbb{V}$.

The key lemma

- $\mathbf{A}$ an n-dimensional Tortkara algebra with $n > \dim(\mathrm{Ann}\,\mathbf{A}) = m \neq 0$.

Lemma 3.9

There exist

- *a unique up to* $\cong$ *Tortkara algebra* $\mathbf{A}'$ *with* $\dim \mathbf{A}' = n - m$,
- *a vector space* $\mathbb{V}$ *with* $\dim \mathbb{V} = m$,
- $\theta \in Z^2_T(\mathbf{A}', \mathbb{V})$ *with* $\mathrm{Ann}\,\mathbf{A}' \cap \mathrm{Ann}\,\theta = 0$,

such that $\mathbf{A} \cong \mathbf{A}'_\theta$ *and* $\mathbf{A}/\,\mathrm{Ann}\,\mathbf{A} \cong \mathbf{A}'$.

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The coordinates of a cocycle

- $\mathbf{A}$ a Tortkara algebra over $\mathbb{C}$;
- $\mathbb{V}$ a $\mathbb{C}$-vector space with basis $e_1, \ldots, e_s$;
- $\theta \in Z_T^2(\mathbf{A}, \mathbb{V})$.

Remark 3.10

$\theta(x, y) = \sum_{i=1}^{s} \theta_i(x, y) e_i$, where $\theta_i \in Z_T^2(\mathbf{A}, \mathbb{C})$. Moreover, $\mathrm{Ann}\,\theta = \mathrm{Ann}\,\theta_1 \cap \mathrm{Ann}\,\theta_2 \cdots \cap \mathrm{Ann}\,\theta_s$. Further, $\theta \in B^2(\mathbf{A}, \mathbb{V})$ if and only if $\theta_i \in B^2(\mathbf{A}, \mathbb{C})$ for all i.

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Non-split extensions

- **A** a Tortkara algebra.

Definition 3.11

A central extension $\mathbf{A}'$ of $\mathbf{A}$ is called *non-split*, if it cannot be represented as the direct sum $\mathbf{A} = I \oplus \mathbb{C}x$.

- $\theta \in Z_T^2(\mathbf{A}, \mathbb{V})$;
- $\mathrm{Ann}\,\theta \cap \mathrm{Ann}\,\mathbf{A} = 0$.

Proposition 3.12

$\mathbf{A}_\theta$ is non-split if and only if $[\theta_1], [\theta_2], \ldots, [\theta_s]$ are linearly independent in $H_T^2(\mathbf{A}, \mathbb{C})$.

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Grassmannian

- $\mathbb{V}$ an n-dimensional vector space over $\mathbb{C}$;
- $1 \leq k \leq n$.

Definition 3.13

The *Grassmannian* $G_k(\mathbb{V})$ is the set of all k-dimensional subspaces of $\mathbb{V}$.

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The description of non-split extensions

- $\mathbf{A}$ a Tortkara algebra over $\mathbb{C}$;
- $\mathbb{V}$ a $\mathbb{C}$-vector space with basis $e_1, \ldots, e_s$.

Definition 3.14

Let $E(\mathbf{A}, \mathbb{V})$ be the set of all *non-split s-dimensional* central extensions of $\mathbf{A}$ by $\mathbb{V}$.

Definition 3.15

Denote by $T_s(\mathbf{A})$ the set of all $\langle [\theta_1], [\theta_2], \ldots, [\theta_s] \rangle \in G_s(H^2(\mathbf{A}, \mathbb{C}))$ such that $\cap_{i=1}^{s} \operatorname{Ann} \theta_i \cap \operatorname{Ann} \mathbf{A} = 0$.

Proposition 3.16

The set $E(\mathbf{A}, \mathbb{V})$ coincides, up to isomorphism, with $\mathbf{A}_\theta$, where $\theta(x, y) = \sum_{i=1}^{s} \theta_i(x, y) e_i$ and $\langle [\theta_1], [\theta_2], \ldots, [\theta_s] \rangle \in T_s(\mathbf{A})$.

The action of $\operatorname{Aut}\mathbf{A}$ on $G_s(H^2(\mathbf{A},\mathbb{C}))$

- $\mathbf{A}$ a Tortkara algebra over $\mathbb{C}$;
- $\phi \in \operatorname{Aut}\mathbf{A}$;
- $W = \langle [\theta_1], [\theta_2], \ldots, [\theta_s] \rangle \in G_s(H^2(\mathbf{A},\mathbb{C}))$.

Proposition 3.17

Put $\phi W = \langle [\phi\theta_1], [\phi\theta_2], \ldots, [\phi\theta_s] \rangle$, *where* $\phi\theta(x,y) = \theta(\phi(x),\phi(y))$.

This defines a natural action of $\operatorname{Aut}\mathbf{A}$ on $G_s(H^2(\mathbf{A},\mathbb{C}))$.

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The isomorphism problem for central extensions

- $\mathbf{A}_\theta, \mathbf{A}_\vartheta \in E(\mathbf{A}, \mathbb{V})$;
- $\theta(x,y) = \sum_{i=1}^{s} \theta_i(x,y)e_i$, $\vartheta(x,y) = \sum_{i=1}^{s} \vartheta_i(x,y)e_i$.

Lemma 3.18

The Tortkara algebras $\mathbf{A}_\theta$ and $\mathbf{A}_\vartheta$ are isomorphic if and only if

$$\mathrm{Orb}\,\langle[\theta_1],[\theta_2],\ldots,[\theta_s]\rangle = \mathrm{Orb}\,\langle[\vartheta_1],[\vartheta_2],\ldots,[\vartheta_s]\rangle.$$

Corollary 3.19

There is a one-to-one correspondence between the set of $\mathrm{Aut}\,\mathbf{A}$-orbits on $T_s(\mathbf{A})$ and the set of isomorphism classes of $E(\mathbf{A}, \mathbb{V})$.

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The Procedure

- $\mathbf{A}'$ a (nilpotent) Tortkara algebra of dimension $n - s$.

Corollary 3.20

In order to classify all n-dimensional central extensions of $\mathbf{A}'$,

1. *determine* $T_s(\mathbf{A}')$ *and* $\mathrm{Aut}\,(\mathbf{A}')$;
2. *determine the set of* $\mathrm{Aut}\,(\mathbf{A}')$*-orbits on* $T_s(\mathbf{A}')$;
3. *for each orbit, construct the Tortkara algebra corresponding to one of its representatives.*

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Malcev algebras

Definition 3.21

An algebra $\mathbf{A}$ is *Malcev*, if it satisfies

1. $xy = -yx$;
2. $(wy)(xz) = ((wx)y)z + ((xy)z)w + ((yz)w)x + ((zw)x)y$.

All nilpotent Malcev algebras of dimension ≤ 6 over a field of characteristic different from 2 were classified in [4]. It turns out that all 5-dimensional nilpotent Malcev algebras over $\mathbb{C}$ are Tortkara.

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Central extensions of Tortkara-Malcev algebras

- **A** a nilpotent Tortkara algebra.

Remark 3.22

If **A** is non-Malcev, then any central extension of **A** is non-Malcev. But if **A** is Malcev, then **A** may have extensions which are Malcev algebras. More precisely, let $\theta \in Z^2_T(\mathbf{A}, \mathbb{V})$. Then $\mathbf{A}_\theta$ is Malcev if and only if

$$\theta(wy, xz) = \theta((wx)y, z) + \theta((xy)z, w) + \theta((yz)w, x) + \theta((zw)x, y). \quad (1)$$

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Tortkara-Malcev cohomologies

- **A** a nilpotent Tortkara-Malcev algebra.

Definition 3.23

Define $Z^2_{TM}(\mathbf{A}, V)$ to be the subspace of all $\theta \in Z^2_T(\mathbf{A}, \mathbb{V})$ such that θ satisfies (1) for all $x, y, z, w \in \mathbf{A}$. Observe that $B^2(\mathbf{A}, V) \subseteq Z^2_{TM}(\mathbf{A}, V)$. Let $H^2_{TM}(\mathbf{A}, V) = Z^2_{TM}(\mathbf{A}, V) / B^2(\mathbf{A}, V)$. Then $H^2_{TM}(\mathbf{A}, V)$ is a subspace of $H^2_T(\mathbf{A}, V)$.

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Non-Malcev Tortkara extensions

- **A** a nilpotent Tortkara-Malcev algebra;

Definition 3.24

Define

$$\mathcal{U}_s(\mathbf{A}) = T_s(\mathbf{A}) \setminus G_s(H^2_{TM}(\mathbf{A}, \mathbb{C})).$$

Proposition 3.25

The non-Malcev Tortkara extensions of **A** *are exactly the Tortkara algebras corresponding to the representatives of* $\mathrm{Aut}\,\mathbf{A}$*-orbits on* $\mathcal{U}_s(\mathbf{A})$.

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The Modified Procedure

- $\mathbf{A}'$ a nilpotent Tortkara algebra of dimension $n - s$.

1. If $\mathbf{A}'$ is non-Malcev, then apply the Procedure.
2. Otherwise, do the following:
   1. determine $\mathcal{U}_s(\mathbf{A}')$ and $\mathrm{Aut}\,\mathbf{A}'$;
   2. determine the set of $\mathrm{Aut}\,\mathbf{A}'$-orbits on $\mathcal{U}_s(\mathbf{A}')$;
   3. for each orbit, construct the Tortkara algebra corresponding to one of its representatives.

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6-dimensional nilpotent Tortkara algebras

Notations

- **A** a nilpotent Tortkara algebra with basis $e_1, e_2, \ldots, e_n$.

Definition 4.1

Denote by Δ_{ij} the skew-symmetric bilinear form on **A** with values in $\mathbb{C}$, such that $\Delta_{ij}(e_l, e_m) = 0$, if $\{i,j\} \neq \{l,m\}$.

Remark 4.2

The set $\{\Delta_{ij} : 1 \leq i < j \leq n\}$ is a basis of the linear space of skew-symmetric bilinear forms on **A**.

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Central extensions of 4-dimensional nilpotent Tortkara algebras

A	multiplication table	$H^2_{TM}(\mathbf{A})$	$H^2_T(\mathbf{A})$
$\mathbb{T}^4_{01}$	$e_1e_2 = e_3$	$\langle[\Delta_{13}], [\Delta_{14}], [\Delta_{23}], [\Delta_{24}], [\Delta_{34}]\rangle$	$H^2_{TM}(\mathbb{T}^4_{01})$
$\mathbb{T}^4_{02}$	$e_1e_2 = e_3, e_1e_3 = e_4$	$\langle[\Delta_{14}], [\Delta_{23}]\rangle$	$H^2_{TM}(\mathbb{T}^4_{02}) \oplus \langle[\Delta_{24}]\rangle$

Corollary 4.3

Every Tortkara central extension of $\mathbb{T}^4_{01}$ *is a Malcev algebra. There is only one 5-dimensional Tortkara non-Malcev central extension of* $\mathbb{T}^4_{02}$*:*

$$\mathbb{T}^5_{10} \quad : \quad e_1e_2 = e_3, \quad e_1e_3 = e_4, \quad e_2e_4 = e_5.$$

The 6-dimensional Tortkara non-Malcev central extensions of $\mathbb{T}^4_{02}$ *are*

$$\begin{array}{lllllll} \mathbb{T}^6_{01} & : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_1e_4 = e_5, & e_2e_3 = e_5, & e_2e_4 = e_6; \\ \mathbb{T}^6_{02} & : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_2e_3 = e_5, & e_2e_4 = e_6; & \\ \mathbb{T}^6_{03} & : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_1e_4 = e_5, & e_2e_4 = e_6. & \end{array}$$

The 5-dimensional nilpotent Tortkara algebras

A	multiplication table	$H^2_{TM}(\mathbf{A})$	$H^2_T(\mathbf{A})$
$\mathbb{T}^5_{01}$	$e_1e_2 = e_3$	$\left\langle \begin{array}{l}[\Delta_{13}], [\Delta_{14}], [\Delta_{15}], [\Delta_{23}], [\Delta_{24}],\\ [\Delta_{25}], [\Delta_{34}], [\Delta_{35}], [\Delta_{45}]\end{array} \right\rangle$	$H^2_{TM}(\mathbb{T}^5_{01})$
$\mathbb{T}^5_{02}$	$e_1e_2 = e_3, e_1e_3 = e_4$	$\left\langle [\Delta_{14}], [\Delta_{15}], [\Delta_{23}], [\Delta_{25}], [\Delta_{35}] \right\rangle$	$H^2_{TM}(\mathbb{T}^5_{02}) \oplus \langle [\Delta_{24}], [\Delta_{45}] \rangle$
$\mathbb{T}^5_{03}$	$e_1e_2 = e_4, e_1e_3 = e_5$	$\left\langle \begin{array}{l}[\Delta_{14}], [\Delta_{15}], [\Delta_{23}], [\Delta_{24}],\\ [\Delta_{25}], [\Delta_{34}], [\Delta_{35}]\end{array} \right\rangle$	$H^2_{TM}(\mathbb{T}^5_{03})$
$\mathbb{T}^5_{04}$	$e_1e_2 = e_3, e_1e_3 = e_4,$ $e_2e_3 = e_5$	$\left\langle [\Delta_{14}], [\Delta_{15}] + [\Delta_{24}], [\Delta_{25}] \right\rangle$	$H^2_{TM}(\mathbb{T}^5_{04}) \oplus \langle [\Delta_{15}] \rangle$
$\mathbb{T}^5_{05}$	$e_1e_2 = e_5, e_3e_4 = e_5$	$\left\langle \begin{array}{l}[\Delta_{12}], [\Delta_{13}], [\Delta_{14}], [\Delta_{15}], [\Delta_{23}],\\ [\Delta_{24}], [\Delta_{25}], [\Delta_{35}], [\Delta_{45}]\end{array} \right\rangle$	$H^2_{TM}(\mathbb{T}^5_{05})$
$\mathbb{T}^5_{06}$	$e_1e_2 = e_3, e_1e_4 = e_5,$ $e_2e_3 = e_5$	$\left\langle [\Delta_{13}], [\Delta_{14}], [\Delta_{24}], [\Delta_{25}], [\Delta_{34}] \right\rangle$	$H^2_{TM}(\mathbb{T}^5_{06}) \oplus \langle [\Delta_{15}], [\Delta_{45}] \rangle$
$\mathbb{T}^5_{07}$	$e_1e_2 = e_3, e_3e_4 = e_5$	$\left\langle [\Delta_{13}], [\Delta_{14}], [\Delta_{23}], [\Delta_{24}] \right\rangle$	$H^2_{TM}(\mathbb{T}^5_{07})$
$\mathbb{T}^5_{08}$	$e_1e_2 = e_3, e_1e_3 = e_4,$ $e_1e_4 = e_5$	$\left\langle [\Delta_{15}], [\Delta_{23}] \right\rangle$	$H^2_{TM}(\mathbb{T}^5_{08}) \oplus \langle [\Delta_{24}], [\Delta_{25}] \rangle$
$\mathbb{T}^5_{09}$	$e_1e_2 = e_3, e_1e_3 = e_4,$ $e_1e_4 = e_5, e_2e_3 = e_5$	$\left\langle [\Delta_{14}], [\Delta_{15}] + [\Delta_{24}] \right\rangle$	$H^2_{TM}(\mathbb{T}^5_{09}) \oplus \langle [\Delta_{15}], [\Delta_{25}] \rangle$
$\mathbb{T}^5_{10}$	$e_1e_2 = e_3, e_1e_3 = e_4,$ $e_2e_4 = e_5$	—	$\langle [\Delta_{14}], [\Delta_{23}], [\Delta_{34}] + [\Delta_{15}] \rangle$

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Central extensions of 5-dimensional nilpotent Tortkara algebras

Proposition 4.4

The 6-dimensional non-split Tortkara non-Malcev central extensions of $\mathbb{T}^5_{02}$ are:

$$\begin{array}{lllll}
\mathbb{T}^6_{04} : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_1e_5 = e_6, & e_2e_4 = e_6; \\
\mathbb{T}^6_{05} : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_2e_3 = e_6, & e_4e_5 = e_6; \\
\mathbb{T}^6_{06} : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_2e_4 = e_6, & e_3e_5 = e_6; \\
\mathbb{T}^6_{07} : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_4e_5 = e_6. &
\end{array}$$

The 6-dimensional non-split Tortkara non-Malcev central extensions of $\mathbb{T}^5_{04}$ are:

$$\begin{array}{lllllll}
\mathbb{T}^6_{08} : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_1e_4 = e_6, & e_1e_5 = -e_6, & e_2e_3 = e_5, & e_2e_4 = e_6; \\
\mathbb{T}^6_{09}(\alpha) : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_1e_5 = (\alpha+1)e_6, & e_2e_3 = e_5, & e_2e_4 = \alpha e_6, &
\end{array}$$

where $\mathbb{T}^6_{09}(\alpha) \cong \mathbb{T}^6_{09}(\beta)$ if and only if $\beta = -\alpha - 1$.
The 6-dimensional non-split Tortkara non-Malcev central extensions of $\mathbb{T}^5_{06}$ are:

$$\begin{array}{llllll}
\mathbb{T}^6_{10} : & e_1e_2 = e_3, & e_1e_3 = e_6, & e_1e_4 = e_5, & e_2e_3 = e_5, & e_4e_5 = e_6; \\
\mathbb{T}^6_{11} : & e_1e_2 = e_3, & e_1e_4 = e_5, & e_1e_5 = e_6, & e_2e_3 = e_5; & \\
\mathbb{T}^6_{12} : & e_1e_2 = e_3, & e_1e_4 = e_5, & e_1e_5 = e_6, & e_2e_3 = e_5, & e_2e_4 = e_6; \\
\mathbb{T}^6_{13} : & e_1e_2 = e_3, & e_1e_4 = e_5, & e_1e_5 = e_6, & e_2e_3 = e_5, & e_3e_4 = e_6; \\
\mathbb{T}^6_{14} : & e_1e_2 = e_3, & e_1e_4 = e_5, & e_2e_3 = e_5, & e_4e_5 = e_6. &
\end{array}$$

Central extensions of 5-dimensional nilpotent Tortkara algebras

Proposition 4.5

The 6-dimensional non-split Tortkara non-Malcev central extensions of $\mathbb{T}^5_{08}$ *are:*

$$\begin{array}{lllllll} \mathbb{T}^6_{15} & : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_1e_4 = e_5, & e_1e_5 = e_6, & e_2e_4 = e_6; \\ \mathbb{T}^6_{16} & : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_1e_4 = e_5, & e_2e_5 = e_6. & \end{array}$$

The 6-dimensional non-split Tortkara non-Malcev central extensions of $\mathbb{T}^5_{09}$ *are:*

$$\begin{array}{llllllll} \mathbb{T}^6_{17} & : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_1e_4 = e_5, & e_2e_3 = e_5, & e_2e_5 = e_6; & \\ \mathbb{T}^6_{18}(\alpha) & : & e_1e_2 = e_3, & e_1e_3 = e_4, & e_1e_4 = e_5, & e_1e_5 = (\alpha+1)e_6, & e_2e_3 = e_5, & e_2e_4 = \alpha e_6. \end{array}$$

There is only one 6-dimensional non-split Tortkara non-Malcev central extension of $\mathbb{T}^5_{10}$*:*

$$\mathbb{T}^6_{19} \quad : \quad e_1e_2 = e_3, \quad e_1e_3 = e_4, \quad e_1e_5 = e_6, \quad e_2e_4 = e_5, \quad e_3e_4 = e_6.$$

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The main result

Theorem 4.6 (Gorshkov, Kaygorodov and Khrypchenko [3])

Let $\mathbb{T}$ *be a 6-dimensional nilpotent non-Malcev Tortkara algebra over* $\mathbb{C}$*. Then* $\mathbb{T}$ *is isomorphic to one of the algebras* $\mathbb{T}^6_{00} - \mathbb{T}^6_{19}$*. In particular, only one of this algebras is not non-metabelian:* $\mathbb{T}^6_{19}$*.*

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