Centro de Matemática da Universidade do Porto
Analysis Seminar

A collection of classical three-fold symmetric 2-orthogonal polynomials
by

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Abstract

The three-fold symmetry of a polynomial sequence means that $P_n(\omega^k x) = \omega^{nk} P_n(x)$ for all $n \in \mathbb{N}$ with $\omega = \mathrm{e}^{\frac{2i\pi}{3}}$ and $k = 0, 1$ and 2. The 2-orthogonal polynomials $\{P_n(x)\}_{n\in\mathbb{N}}$ with respect to a vector of measures (μ_0, μ_1) are multiple orthogonal polynomials of type II $\{\widehat{P}_{\vec{n}}(x)\}_{\vec{n}\in\mathbb{N}^2}$ whose index lies on the step line. So, we can assume $P_{2n}(x) = \widehat{P}_{n,n}(x)$ and $P_{2n+1}(x) = \widehat{P}_{n,n+1}(x)$, for all $n \in \mathbb{N}$. A three-fold symmetric 2-orthogonal polynomials satisfy the third-order recurrence relation

$$P_{n+1}(x) = xP_n(x) - \gamma_{n-1} P_{n-2}(x)$$

with initial conditions $P_k(x) = x^k$ for $k = 0, 1, 2$. The focus of this talk is on the characterisation of all the three-fold symmetric 2-orthogonal polynomials such that the sequence of its derivatives is again 2-orthogonal. This is basically an extension of the Hahn's property into the context of 2-orthogonality. Recall that within the (standard) orthogonality context, only the classical polynomials of Hermite, Laguerre, Bessel and Jacobi share this property. So, in a way, we will be discussing an extension of the classical polynomials in the 2-orthogonality context. Among the properties under analysis are the location and asymptotic behaviour of the zeros alongside with the orthogonality measures supported on a three-star of the complex plane. We will show that there are essentially three distinct families of three-fold symmetric 2-orthogonal Hahn-classical polynomials, whose weight functions are described via Airy function, confluent hypergeometric functions and hypergeometric functions, respectively. This is a joint work with Walter Van Assche.